SCORING GUIDELINES

Question 3

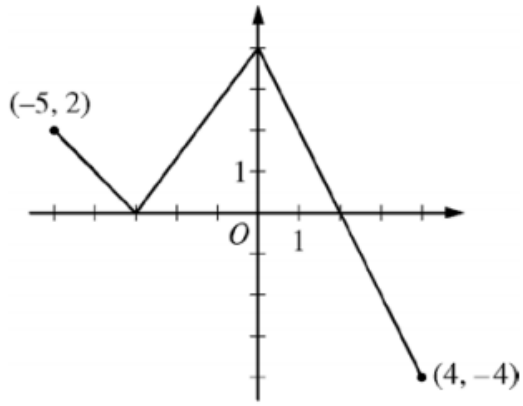
The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.



Graph of f

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$

$$h'(3) = \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$$

$$= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3}$$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

(d) $p'(x) = f'\left(x^2 - x\right)(2x - 1)$

$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$

$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$