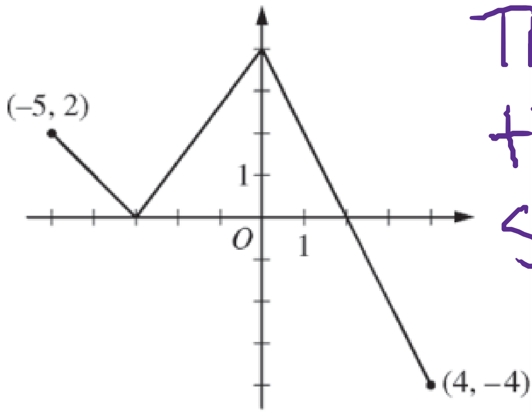


Graph of f

The slope of this linear segment is

$$\frac{-4-4}{4-0} = \frac{-8}{4} = -2$$

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

To find $g(3)$, substitute 3 in for x to get $g(3) = \int_{-3}^{3} f(t)\,dt$.

To find the integral given a graph, we need to find the area under the curve from $x=-3$ to $x=3$.

Triangle from $-3 \le x \le 2$

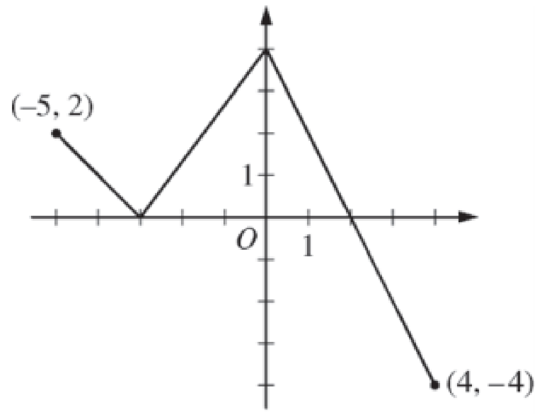
$A = \frac{1}{2} \cdot 5 \cdot 4 = 10$

Above x-axis

Triangle from $2 \le x \le 3$

B/c of slope of -2 and using either $(0,4)$ or $(4,-4)$, when $x=3$, $y=-2$.

$A = \frac{1}{2} \cdot 1 \cdot 2 = 1$ Below x-axis



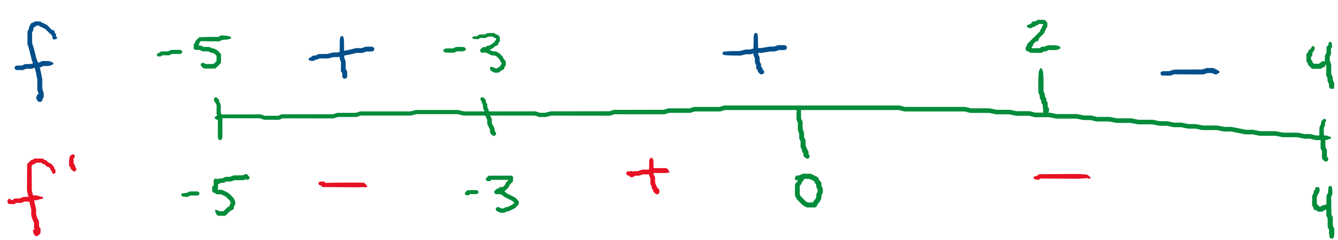
Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

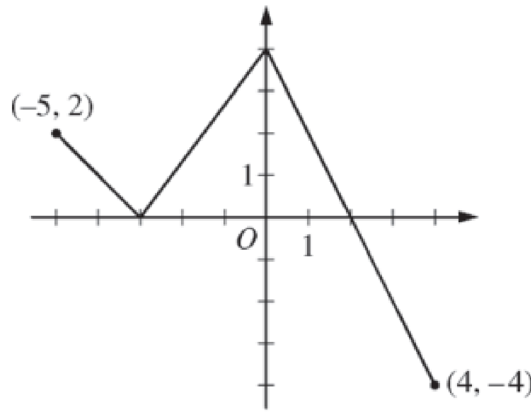
(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

g is increasing when its derivative $g' = f$ is positive and g is concave down when its second derivative $g'' = f'$ is negative.

By looking at the graph, f is positive on the intervals $(-5,-3)$ and $(-3,2)$. f is decreasing (f' is negative) on the intervals $(-5,-3)$ and $(0,4)$.



Remember, $g(3)$ was found in Part (a)



Graph of f

Also remember, $g'(x) = f(x)$ $\therefore$ $g'(3) = f(3)$, which was also found in Part (a).

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

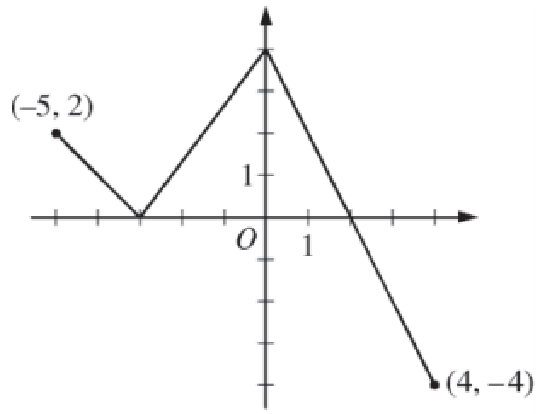
(c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

First thing to recognize here is that in order to find $h'(x)$, we need to use Quotient Rule for Derivatives.

$$h'(x) = \frac{5x \cdot g'(x) - g(x) \cdot 5}{(5x)^2}$$

Next, substitute 3 in for x to get

$$h'(3) = \frac{5(3) \cdot g'(3) - g(3) \cdot 5}{(5 \cdot 3)^2}$$



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

We need 2 things for a tangent line:
1. Point 2. Slope (Derivative)
But, this problem only wants the slope.
To find the slope/derivative of $p(x)$, we need to use the Chain Rule.

$$p'(x) = f'(x^2 - x) \cdot (2x - 1)$$

Then, substitute -1 in for x to get

$$p'(-1) = f'((-1)^2 - (-1)) \cdot (2(-1) - 1)$$

$$p'(-1) = f'(2) \cdot (-3)$$

Look at the slope of the graph, which was found in Part (a).