*Keep in mind that velocity and acceleration can be negative.

No Calculator Allowed

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

The function/table above is the velocity of a train. This problem wants us to find average acceleration. Acceleration is the derivative (or rate of change) of velocity.

So, in this problem, we are finding the average rate of change from $2 \le t \le 8$ for the given table.

Remember, average rate of change is found by $\frac{y_2 - y_1}{x_2 - x_1}$.

We just use these two points.

$$\frac{-120 - 100}{8 - 2}$$

Be sure to include units.

No Calculator Allowed

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	-120	-150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

This is one of those common math sense things... which means we need a theorem.

Because when $t = 5$, $v(t) = 40$ and when $t = 8$, $v(t) = -120$, then the velocity has to be -100 at some time t between $t = 5$ and $t = 8$ as long as the function $v(t)$ is continuous.

Remember, if a function is differentiable then it must also be continuous (but not the other way around).

The theorem that can be used here is Intermediate Value Theorem, IVT.

No Calculator Allowed

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	–120	–150

West is negative direction.

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

$s(2) = 300$ East is positive direction.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

Position is the antiderivative of velocity. In order to find position given velocity, we need to take an integral

To find the position of the train at $t=12$, we need to use the given position at $t=2$ and add that to the displacement (integral) from $t=2$ to $t=12$.

$$s(12) = s(2) + \int_2^{12} v(t)\,dt.$$

To find $\int_2^{12} v(t)\,dt$ given a table, we need to use trapezoids to approximate the area under a curve.

Remember, area of a trapezoid is $\frac{1}{2}(b_1+b_2)h$ or in this case $\frac{1}{2}(v_1+v_2)\Delta t$.

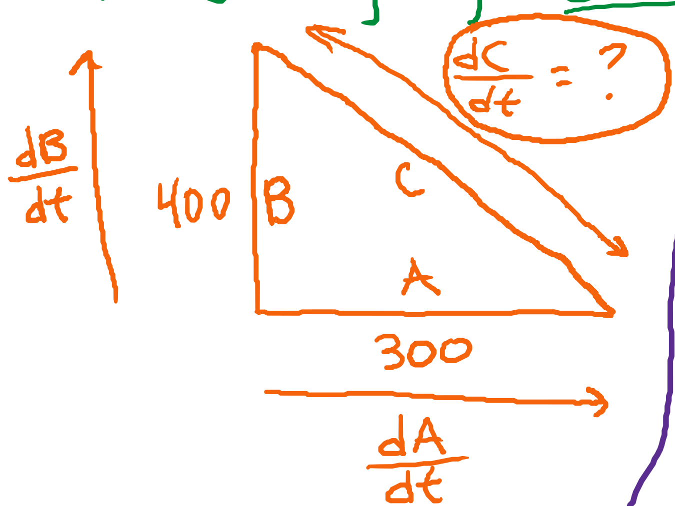
No Calculator Allowed

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

From Part (c), train A is 300 m east at $t=2$. We know train B travels north and we want to find the rate at which the distance is changing. Draw a picture.



Equation: $A^2 + B^2 = C^2$

* There is no constant to substitute in right away.
* $A = 300$ and $B = 400$ are when $t = 2$. These values can be plugged in after taking the derivative.

$$\frac{d}{dt}\left[A^2 + B^2 = C^2\right]$$

$$\cancel{2}A\frac{dA}{dt} + \cancel{2}B\frac{dB}{dt} = \cancel{2}C\frac{dC}{dt}$$

* $\frac{dA}{dt} = v_A(t)$

At $t = 2$, $\frac{dA}{dt} = 100$ by using the table above.

* Solve for C at $t=2$ by using $A=300$ and $B=400$ and $A^2+B^2=C^2$.

$$300^2+400^2=C^2$$

$$C=500$$

* $\frac{dB}{dt}=V_B(t)$

At $t=2$, we use the equation for $V_B(t)$.

$$V_B(2)=-5(2)^2+60(2)+25$$

$$=-20+120+25$$

$$\frac{dB}{dt}=125$$

$$300(100)+400(125)=500\frac{dC}{dt}$$

Solve for $\frac{dC}{dt}$ which represents the rate (or velocity) at which the distance between the two trains is changing.