

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (a) Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

f has a maximum or a minimum when $f' = 0$ and f' changes signs.

$f' = 0$ at both $x = -1$ and $x = 1$, but f' does not change signs at $x = -1$. A relative minimum exists when f' changes from negative to positive, which happens at $x = 1$.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
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5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (b) Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.

f'' is the derivative of f' and we know next to nothing about f'' . The setup of this problem leads us to a "theorem is needed".

Since we are being asked to find something out about a derivative, we can use Mean Value Theorem (which states that at some value c for a differentiable function, the average rate of change must be equal to the instantaneous rate of change)

$$\text{AROC} = \text{IROC} \quad \text{Use the points } (-1, 0) \text{ and } (1, 0) \text{ for } f'(x).$$

$$\frac{f'(b) - f'(a)}{b-a} = f''(c)$$

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
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5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

Before substituting 3 in for x , we need to find $h'(x)$. In order to find $h'(x)$ from the given $h(x)$, we need to use the Chain Rule for Derivatives.

Remember, the derivative of $\ln u$ is $\frac{u'}{u}$.

$$h'(x) = \frac{f'(x)}{f(x)} \quad (\text{Chain Rule has been used here.})$$

Now, find $h'(3)$ by plugging in 3 for x and using the table above.

$$h'(3) = \frac{f'(3)}{f(3)} = \frac{\frac{1}{2}}{7}$$

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
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5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

(d) Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.

The first thing we see here is that we have a function within a function. For a derivative, that means we use Chain Rule. But for an integral, that means we need to use U-Substitution.

$$\begin{aligned} u &= g(x) \\ du &= g'(x) dx \end{aligned}$$

Everything we need to use is already in the problem.

2 Ways to Solve

Change interval to be in terms of u .

$$u(-2) = g(-2) = -1$$

$$u(3) = g(3) = 1$$



Take the integral and substitute $g(x)$ back in for u .

$$\int_{-2}^3 f'(u) du$$



$$\begin{array}{ccc}
 \int_{-1}^1 f'(u) du & & f(u) \\
 \downarrow & & \downarrow \\
 f(u) \Big|_{-1}^1 & & f(g(x)) \Big|_{-2}^3 \\
 \downarrow & & \downarrow \\
 f(1) - f(-1) & & f(g(3)) - f(g(-2)) \\
 & & \downarrow \\
 * \text{ Use the} & & f(1) - f(-1) \\
 \text{table} & & \\
 \text{above to} & & \\
 \text{solve.} & &
 \end{array}$$