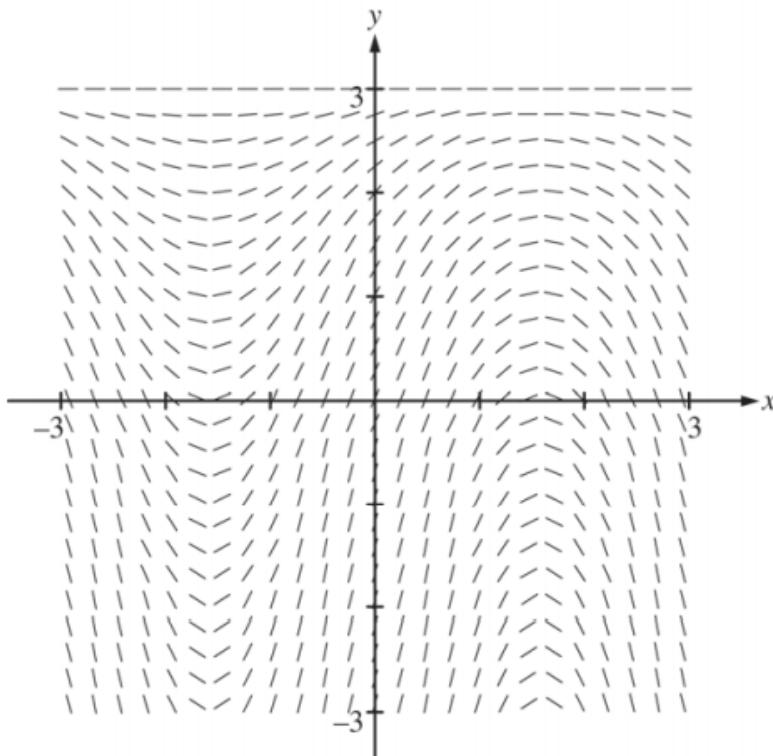


6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.
- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



Remember that a slope field is just that ... a field of slopes at different points that act as a force field telling the curve where to go.

The first thing to do is to start at the given point of $f(0) = 1$, then draw a curve that follows the path of the slopes around it.

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

The two things needed to write an equation for a tangent line are: 1. Point and 2. Slope
 The point $(0, 1)$ is given to us, so we just need to find the slope (derivative) at this point. We are given $\frac{dy}{dx}$, which is the derivative, and just need to substitute 0 in for x and 1 in for y .

$$\frac{dy}{dx} = (3 - 1) \cdot \cos 0 \Rightarrow \frac{dy}{dx} = 2$$

$$y - 1 = 2(x - 0) \Rightarrow y - 1 = 2x \quad \text{or}$$

$$y = 2x + 1$$

Use this equation to estimate $f(0.2)$ by substituting 0.2 in for x in $y = 2x + 1$

$f(0.2) \approx 2(0.2) + 1$

The approximate symbol is needed.

6. Consider the differential equation $\frac{dy}{dx} = (3-y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

Remember, a particular solution is the equation that has a derivative to match our differential equation. So, since we are given $\frac{dy}{dx}$, we need to take the integral to find the general solution (which means we need the $+C$) and then solve for C by using the point $(0,1)$.

In order to integrate, we need to separate the variables first.

$$\int \frac{1}{3-y} dy = \int \cos x dx$$

$$u = 3 - y \\ du = -1 dy \Rightarrow \int \frac{1}{u} \cdot -du = \int \cos x dx$$

$$-\int \frac{1}{u} du = \int \cos x dx$$

$$-\ln|u| = \sin x + C$$

* Since the point we are using is $(0, 1)$,

by plugging 1 into $3-y$,

we get 2 ,

which is positive.

$$-\ln|3-y| = \sin x + C$$

$$\frac{-\ln(3-y)}{-1} = \frac{\sin x + C}{-1}$$

$$e^{\ln(3-y)} = e^{-\sin x + C}$$

$$3-y = e^{-\sin x} \cdot e^C$$

$$3-y = Ce^{-\sin x}$$

$$-y = Ce^{-\sin x} - 3$$

$$y = Ce^{-\sin x} + 3$$

$$1 = Ce^{-\sin 0} + 3$$

$$1 = Ce^0 + 3$$

$$1 = C(1) + 3$$

$$-2 = C$$

$$y = -2e^{-\sin x} + 3$$