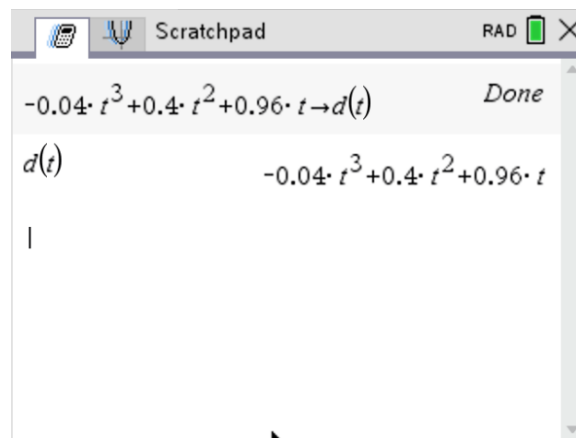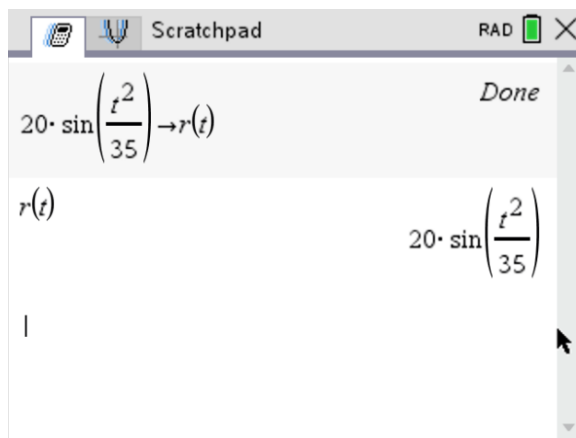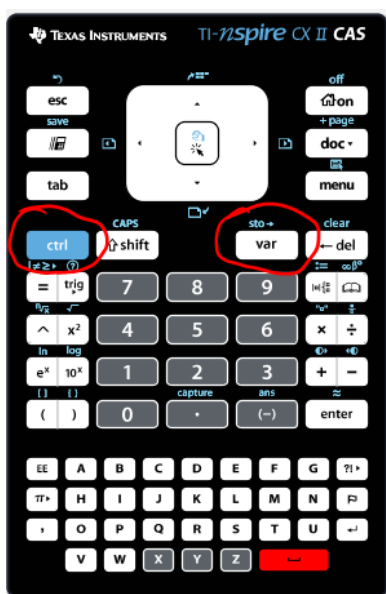1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\dfrac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

Before starting this problem, it will help us save time by storing each function $R(t)$ and $D(t)$ into our calculator.

I am storing these in function notation because it makes it easier to evaluate $R(t)$ or $D(t)$ at a certain time, t, which we will need to do in part (b).

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\dfrac{t^2}{35}\right)$
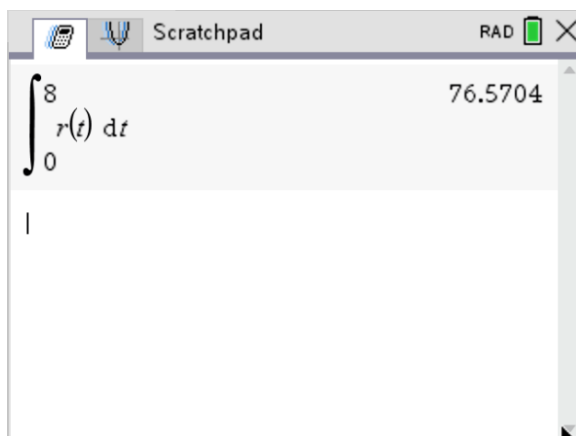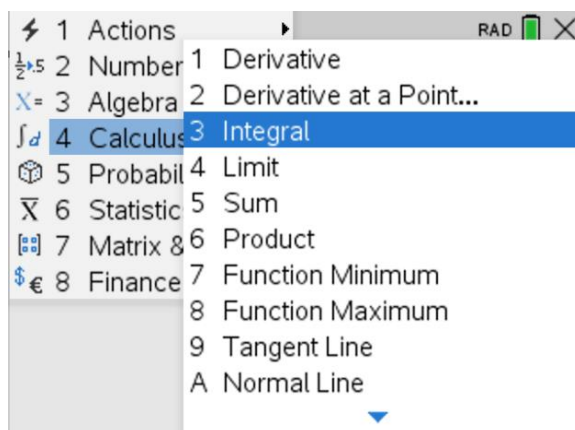
   cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

   (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

This problem wants the total amount of rainwater that has flowed into the pipe. Since we are only concerned about rainwater flowing into the pipe, we only use R(t). Also, R(t) is the rate of rainwater flowing into the pipe. So, to find our total, we take the integral of R(t) on 0≤t≤8.

1 Actions
5 2 Number   1 Derivative
X= 3 Algebra   2 Derivative at a Point...
∫d 4 Calculus 3 Integral
5 Probabil 4 Limit
X̄ 6 Statistic 5 Sum
7 Matrix & 6 Product
$€ 8 Finance   7 Function Minimum
              8 Function Maximum
              9 Tangent Line
              A Normal Line

Scratchpad

$$\int_0^8 r(t)\, dt$$

76.5704

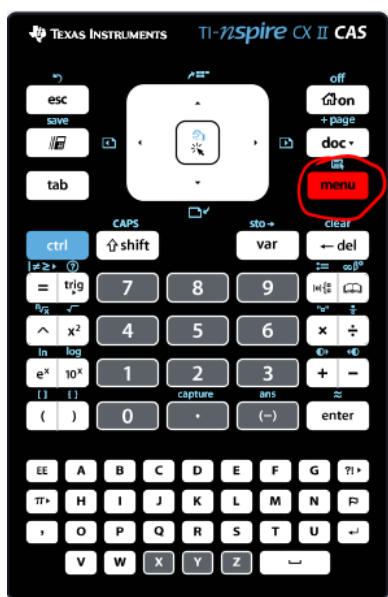1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\dfrac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

For this problem, to see if the amount of water in the pipe is increasing or decreasing, we need to compare $R(t) \Longleftrightarrow$ increase and $D(t) \Longleftrightarrow$ decrease at the time $t = 3$ hours.

Scratchpad	RAD ☐ ✕
$r(3)$	$20 \cdot \sin\left(\dfrac{9}{35}\right)$
$r(3)$	5.08637
$d(3)$	5.4
|	
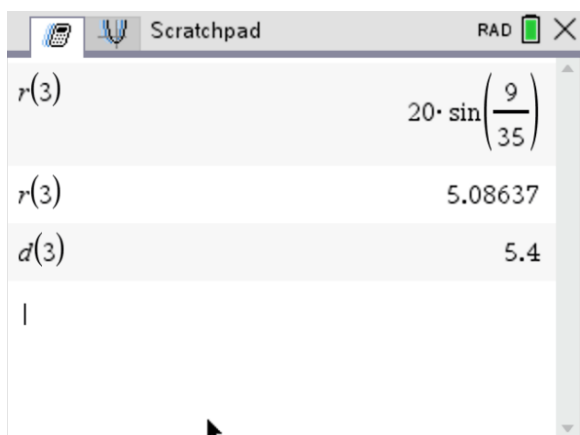
* Press 'ctrl', then 'enter' to get r(3) into decimal form.

TEXAS INSTRUMENTS   TI-nspire CX II CAS

As shown by finding R(3) and D(3) in our calculator, since D(3) has the greater value, the amount of rainwater in the pipe at t = 3 hours is decreasing.
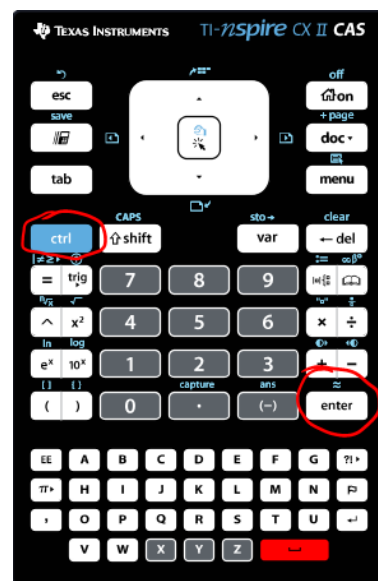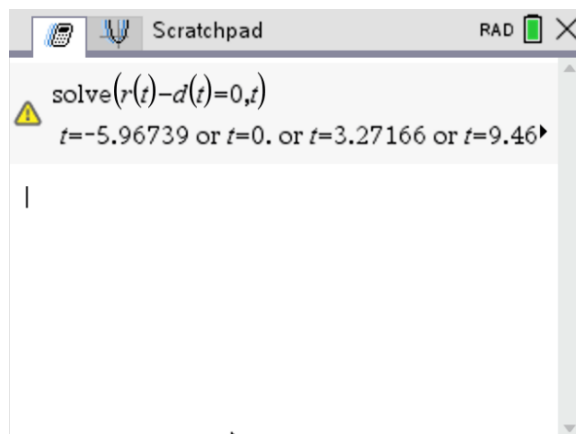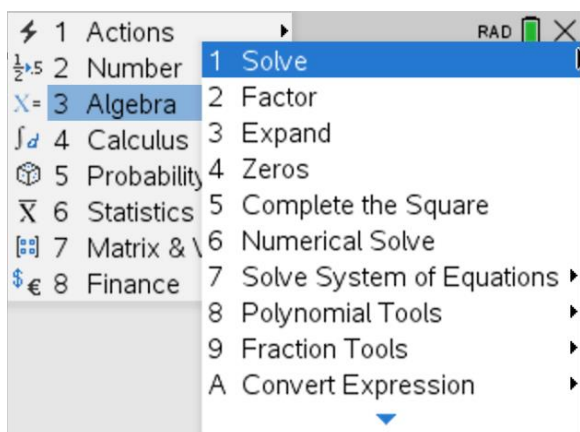
1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\dfrac{t^2}{35}\right)$

cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

A minimum occurs when the derivative of a function equals zero and when the derivative changes from negative to positive. Remember, a derivative is also called a rate of change. In this problem, we are given two rates (two derivatives). So, to find when there is a minimum, we need to find when the rate of increase combined with the rate of decrease equals zero. $R(t) - D(t) = 0$

⚡ 1 Actions	▶	RAD ⬜ ✕
½▸5 2 Number	1 Solve	
X= 3 Algebra	2 Factor	
∫d 4 Calculus	3 Expand	
⬡ 5 Probability	4 Zeros	
X̄ 6 Statistics	5 Complete the Square	
[::] 7 Matrix & \	6 Numerical Solve	
$€ 8 Finance	7 Solve System of Equations ▶	
	8 Polynomial Tools ▶	
	9 Fraction Tools ▶	
	A Convert Expression ▶	
	▼	

	🖩 🦅 Scratchpad	RAD ⬜ ✕
⚠	$\text{solve}(r(t) - d(t) = 0, t)$	
	$t = -5.96739$ or $t = 0$. or $t = 3.27166$ or $t = 9.46$ ▸	
	❘	

We only care about the interval $0 \le t \le 8$, ∴ there are three possible

Answers: $t = 0$, $t = 3.272$, $t = 8$.

There is a minimum at $t = 3.272$ hours because the derivative changes from negative to positive.

Scratchpad — RAD
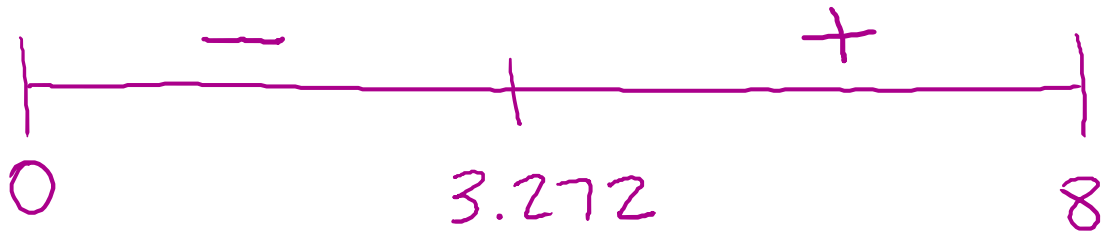
$r(2) - d(2)$	-0.919258
$r(5) - d(5)$	3.30156

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

Again, $R(t) - D(t)$ represents the rate of change at which the rainwater flows through a drainpipe. ∴ the integral $\int_0^x [R(t) - D(t)]\, dt$ represents the total amount of rainwater flowing through the pipe at any time, x.

When writing an integral for this situation, make sure to use the initial condition.

Also, we want our integral to equal 50 because that is the total amount of rainwater the pipe can hold before it starts overflowing. Time is w here.

$$30 + \int_0^w [R(t) - D(t)]\, dt = 50$$