Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

(a) $v'(16) \approx \dfrac{240 - 200}{20 - 12} = 5$ meters/min^2

1 : approximation

(b) $\int_0^{40} |v(t)|\, dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes.

$$\begin{aligned}\int_0^{40} |v(t)|\, dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

3 : { 1 : explanation; 1 : right Riemann sum; 1 : approximation }

(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$B'(5) = 3(25) - 12(5) = 15$ meters/min^2

2 : { 1 : uses $B'(t)$; 1 : answer }

(d) $$\begin{aligned}\text{Avg vel} &= \frac{1}{10}\int_0^{10} \left(t^3 - 6t^2 + 300\right) dt \\ &= \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10} \\ &= \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350 \text{ meters/min}\end{aligned}$$

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

NO CALCULATOR ALLOWED

1 of 2
3A

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

a) $v'(16) \approx \dfrac{v(20)-v(12)}{20-12} = \dfrac{240-200}{8} = \dfrac{40}{8} = \dfrac{20}{4} = 5 \dfrac{m}{min^2}$

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

b) $\int_0^{40} |v(t)|\,dt$ represents the total distance in meters Johanna traveled between times $t=0$ and $t=40$ minutes.

$\int_0^{40} |v(t)|\,dt \approx [12(200) + 8(240) + 4(220) + 16(150)]$

$= 2400 + 1920 + 880 + 2400$

$= 7600$ meters

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(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

c) acceleration $= B'(t) = 3t^2 - 12t$

$B'(5) = 3(5)^2 - 12(5)$

$= 75 - 60 = 15 \frac{m}{min^2}$

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

d) Avg. velocity $= \frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300)\,dt$

$= \frac{1}{10} \cdot \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10}$

$= \frac{1}{10} \cdot \left[\frac{10000}{4} - 2000 + 3000 \right]$

$= \frac{1}{10} [3500] = 350 \frac{m}{min}$

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NO CALCULATOR ALLOWED

3B1

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

$$v'(16) = \frac{v(20) - v(12)}{20 - 12} = \frac{40}{8} = 5$$

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

$\int_0^{40} |v(t)|\, dt$ is the total distance covered by Johanna from $t = 0$ to $t = 40$.

$200(12) + 240(8) + 220(4) + 150(16)$

$= 2400 + 1920 + 880 + 2400$

$= 4800 + 2800 = 7600$

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Continue problem 3 on page 15.

3B2

NO CALCULATOR ALLOWED

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

$B'(t) = 3t^2 - 12t$

$B'(5) = 3 \cdot 25 - 12 \cdot 5$

$= 75 - 60 = 15$ (m/minute)/minute

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

$\frac{1}{10} \int_0^{10} t^3 - 6t^2 + 300 \, dt$ meters/minute

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NO CALCULATOR ALLOWED

3 C1

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

12 8 4 6

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

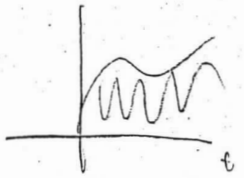
(a) Use the data in the table to estimate the value of $v'(16)$.

$$v'(16) \approx \frac{240-200}{20-12} = \frac{40}{6} = \boxed{\frac{20}{3}}$$

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

$\int_0^{40} |v(t)|$ is the total distance travelled (including when Johanna jogs backwards as positive distance)



positive

total distance $= (12)(200) + 8(240) + 4(+220) + 6(150)$

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NO CALCULATOR ALLOWED

$3C_2$

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

$$a = \frac{dv}{dt} = 3t^2 - 12t$$

$$a(5) = 3(25) - 12(5)$$

$$75 - 60$$

$$= \boxed{15 \text{ meters/minute}^2}$$

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

$a = \frac{dv}{dt}$

$v = \int a \, dt$

$$\text{average velocity} = \frac{\int_0^{10} 3t^2 - 12t}{10 - 0}$$

$$= \left[\frac{t^3 - 6t^2}{10}\right]_0^{10} = \frac{10^3 - 6(10^2)}{10} - 0$$

$$= \frac{1000 - 600}{10}$$

$$= \frac{400}{10} = \boxed{40 \text{ meters/minute}}$$

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Question 3

Overview

In this problem students were given a table of values of a differentiable function v, the velocity of a jogger, in meters per minute, jogging along a straight path for selected values of t in the interval $0 \le t \le 40$. In part (a) students were expected to know that $v'(16)$ can be estimated by the difference quotient $\frac{v(20) - v(12)}{20 - 12}$. In part (b) students were expected to explain that the definite integral $\int_0^{40} |v(t)|\, dt$ gives the total distance jogged, in meters, by Johanna over the time interval $0 \le t \le 40$. Students had to approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the subintervals $[0, 12]$, $[12, 20]$, $[20, 24]$, $[24, 40]$, and values from the table. In part (c) students were given a cubic function B, the velocity of a bicyclist, in meters per minute, riding along the same straight path used by Johanna for $0 \le t \le 10$. Students should have known that $B'(t)$ gives Bob's acceleration at time t. Students were expected to find $B'(t)$ using derivatives of basic functions and then evaluate $B'(5)$. In part (d) students had to set up the definite integral $\frac{1}{10}\int_0^{10} B(t)\, dt$ that gives Bob's average velocity during the interval $0 \le t \le 10$. Students needed to evaluate this integral using basic antidifferentiation and the Fundamental Theorem of Calculus.

Sample: 3A
Score: 9

The response earned all 9 points.

Sample: 3B
Score: 6

The response earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not include "meters," so the explanation point was not earned. The student's Riemann sum and approximation are correct. In part (c) the student's work is correct. In part (d) the student's integral is correct.

Sample: 3C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student attempts to simplify the correct difference quotient but makes an arithmetic error. In part (b) the student did not earn the explanation point because the time interval and the distance units (meters) are not included. The right Riemann sum has exactly one error. The student earned the point because 7 out of the 8 components are correct. The student did not earn the approximation point as a result of an error in the Riemann sum. In part (c) the student's work is correct. In part (d) the student uses $B'(t)$ in the integral instead of $B(t)$.