

| | | | | | |
|-------------------------------|---|-----|-----|------|-----|
| t (minutes) | 0 | 12 | 20 | 24 | 40 |
| $v(t)$ (meters per minute) | 0 | 200 | 240 | -220 | 150 |

3. Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

This problem wants us to find the derivative of $v(t)$ when $t=16$. Remember that a derivative is a rate of change. Since we only have a table, the only way we can estimate $v'(16)$ is to use $t=12$ and $t=20$ since 16 is inbetween 12 and 20 and find the average rate of change.

$$\frac{v(20) - v(12)}{20 - 12} = \frac{240 - 200}{20 - 12} =$$

* Be sure to include appropriate units of measure. The rate of change for meters per minute is meters per minute per minute, or m/min^2 .

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|-------------------------------|---|-----|-----|------|-----|--|
| | | 12 | 8 | 4 | 16 | |
| | | | | | | |
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(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

The integral of velocity is displacement (which can be negative or positive). The integral of absolute value of velocity is total distance traveled (which is always positive, obviously). $\therefore \int_0^{40} |v(t)| dt$ represents the total distance Johanna jogged from $t = 0$ minutes to $t = 40$ minutes.

Next, a right Riemann sum is using rectangles with a height using the right endpoints. Remember, if $v(t)$ is negative, change it to positive since we are finding $\int_0^{40} |v(t)| dt$. *Units = meters

Remember, $A_{\text{rectangle}} = \Delta x \cdot v(t)$
(base \cdot height)

$$12(200) + 8(240) + 4(220) + 16(150) =$$

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- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by

$$B(t) = t^3 - 6t^2 + 300, \text{ where } t \text{ is measured in minutes and } B(t) \text{ is measured in meters per minute.}$$

Find Bob's acceleration at time $t = 5$.

In this problem, we are given Bob's velocity function and asked to find his acceleration. Acceleration is the derivative of velocity.

$$B'(t) = 3t^2 - 12t$$

To find Bob's acceleration at $t = 5$, substitute 5 in for t .

$$B'(5) = 3(5)^2 - 12(5) =$$

*Again, remember that acceleration is measured as meters/minute².

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(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

In this problem, we need to find average velocity and in part (c), we were given the velocity function $B(t) = t^3 - 6t^2 + 300$.
 \therefore we are finding the average value.

Average value is found by $\frac{\text{Integral}}{\text{Interval}}$.

$$\frac{\int_0^{10} (t^3 - 6t^2 + 300) dt}{10 - 0} = \frac{\left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10}}{10}$$

$$= \frac{\left[\frac{10^4}{4} - 2(10)^3 + 300(10) \right] - \left[\frac{0^4}{4} - 2(0)^3 + 300(0) \right]}{10}$$

* Units for average velocity, for velocity period, is meters / minute.