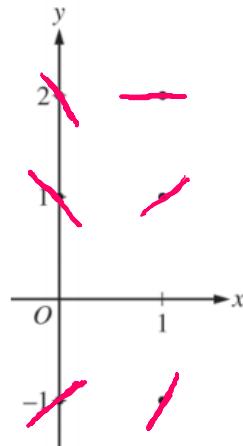


4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



In order to sketch a slope field, simply substitute each point  $(x, y)$  into the differential equation  $\frac{dy}{dx} = 2x - y$ .

$x$	$y$	$\frac{dy}{dx}$
0	2	-2
0	1	-1
0	-1	1
1	2	0
1	1	1
1	-1	3

4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

$\frac{d^2y}{dx^2}$  is the derivative of  $\frac{dy}{dx}$  (with respect to  $x$ ).

$$\frac{d}{dx}[2x-y] = 2 - 1 \cdot \frac{dy}{dx} = 2 - (2x-y)$$

$$\frac{d^2y}{dx^2} = 2 - 2x + y.$$

Remember, the first derivative  $\frac{dy}{dx}$  tells us the slope of our solution curve ( $y =$ ) and the second derivative  $\frac{d^2y}{dx^2}$  tells us the concavity of our solution curve ( $y =$ ).

In Quadrant 2,  $x < 0$  and  $y > 0$ .

Use  $\frac{d^2y}{dx^2}$  to determine if it's (+) Concave Up or (-) Concave Down.

4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.

Remember,  $f$  has a relative maximum or a relative minimum when  $f' = 0$ . If  $f'$  changes from  $(+)$  to  $(-)$ ,  $f$  has a relative maximum. If  $f'$  changes from  $(-)$  to  $(+)$ ,  $f$  has a relative minimum.

Again, the 1<sup>st</sup> thing to figure out is if  $f' = 0$  at  $f(z) = 3$ .

$$f'(x) = \frac{dy}{dx} = 2x - y$$

$$\left. \frac{dy}{dx} \right|_{(z, 3)} = 2(z) - 3 = 1$$

Since  $f' \neq 0$  at  $f(z) = 3$ , there cannot be a relative maximum nor a relative minimum.

4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

## Challenging problem

- (d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.

For this problem, you may think to find the particular solution ( $y =$ ), but we do not have an initial condition.

This problem actually turns into a big substitution problem.

In the form  $y = mx + b$ ,  $m$  represents the slope.  $\frac{dy}{dx}$  also represents slope.

$\therefore m = 2x - y$ . Since we know that

$y = mx + b$ , we substitute into

$$\begin{aligned}m &= 2x - y \Rightarrow m = 2x - (mx + b) \\&\Rightarrow m = 2x - mx - b \\&\Rightarrow 0 = 2x - mx - b - m \\&\Rightarrow 0 = x(2-m) - b - m\end{aligned}$$

Since  $x$  can be any real number, the only way for the above equation to be true is if  $2-m=0$  and additionally if  $-b-m=0$ .

Using  $2 - m = 0$ ,  $m = 2$ .

Then, substitute  $m=2$  into  $-b-m=0$   
and get  $-b-2=0$ .

$$\Rightarrow -b = 2$$

$$b = -2$$