R(t) is similar to velocity here

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

R'(t) is then acceleration here

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

R'(2) is notation for the derivative of R(t) at t=2. R(t) is represented by the table and t=2 is not given, but is inbetween t=1 and t=3. Recall that a derivative is another name for rate of change; and since we cannot find the instantaneous rate of change (IROC) at t=2, we can approximate it using the average rate of change (AROC) from t=1 to t=3.

$$R'(2) \approx \frac{R(3)-R(1)}{3-1} = \frac{950-1190}{2} = \frac{-240}{2} = -120$$

$$\boxed{R'(2) \approx -120 \text{ liters/hr}^2}$$

Makes sense since R(t) is decreasing.

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1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

Remember that the term left Riemann sum is the same as left endpoint rectangles and is used to estimate the area under a curve (or an integral / total amount)

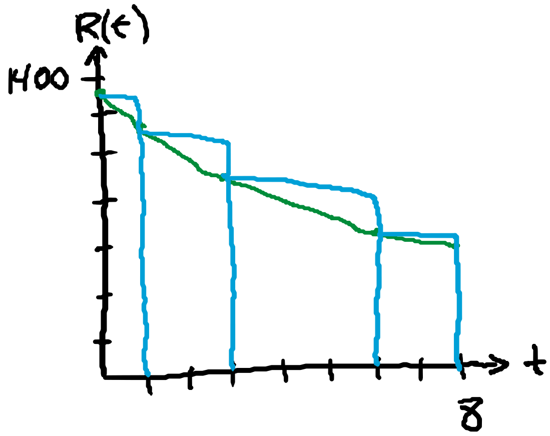
Area of Rectangle = bh or $(\Delta t)(R(t))$

$$1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6)$$

$$1(1340) + 2(1190) + 3(950) + 2(740)$$

$\boxed{8050 \text{ liters}}$

You have a notecard on over/under estimate, but it may help you to visualize a graph of R(t) with left endpoint rectangles.



From this graphical representation, you can see that the rectangles are above the curve making 8050 liters an overestimate.

* Also, because there are left endpoint rectangles on a decreasing function.

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1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

Because we are given the rates of change of the water in the tank with $W(t)$ and $R(t)$, but asked to find the total amount of water in the tank, we need to use integrals (and not forget about our initial condition).

Total amount of water in the tank at $t=8$ (I used $H(t)$ just because)

Total amount of water pumped into the tank, $W(t)$, from $0 \le t \le 8$.

$$H(8) = 50{,}000 + \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt$$

Initial amount of water in the tank at $t=0$.

Total amount of water removed from the tank, $R(t)$, from $0 \le t \le 8$. (Subtracted b/c $R(t)$ is decreasing.)

$$H(8) = 50000 + \int_0^8 \left(2000e^{-t^2/20}\right)dt - 8050$$

$W(t)$

Total from part (b) for $R(t)$

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$$50000+\int_0^8 \left(2000\cdot e^{\frac{-x^2}{20}}\right)dx-8050 \qquad 49786.2$$

$\boxed{\text{49786 liters}}$

Problem wants nearest liter.

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1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

   (d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

At first glace, this problem appears easy with a calculator since we just want to determine when $W(t) = R(t)$. However, since we do not have an actual equation for $R(t)$, this problem leads us to needing to use one of our theorems.

Another way to view $W(t) = R(t)$ is to rewrite it as $W(t) - R(t) = 0$. This helps because now we can think about zero compared to positives and negatives.

$W(0) - R(0) \Rightarrow 2000 - 1340 > 0$

$W(0) = 2000e^{-0^2/20} = 2000e^0 = 2000$

$$W(8) - R(8) \Rightarrow 81.5 - 700 < 0$$

$$W(8) = 2000\, e^{-8^2/20} =$$

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$$2000 \cdot e^{\frac{-8^2}{20}} \quad 2000 \cdot e^{\frac{-16}{5}}$$

$$2000 \cdot e^{\frac{-8^2}{20}} \quad 81.5244$$

Since $W(0) - R(0) > 0$ and $W(8) - R(8) < 0$, then at some t such that $0 \le t \le 8$, $W(t) - R(t) = 0$ because both $W(t)$ and $R(t)$ are continuous.