Question 2

For $t \ge 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$ $v'(4) = -1.164000 < 0$ The particle is slowing down since the velocity and acceleration have different signs.	2 : conclusion with reason
(b) $v(t) = 0 \Rightarrow t = 2.707468$ $v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.	$2 : \begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$
(c) $x(0) = x(4) + \int_4^0 v(t)\,dt$ $= 2 + (-5.815027) = -3.815$	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$
(d) Distance $= \int_0^3 \lvert v(t) \rvert\,dt = 5.301$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

$v(4) = 1 + 2\sin\left(\frac{4^2}{2}\right)$

$v(4) = 2.979$

$v'(t) = 2\cos\left(\frac{t^2}{2}\right)(t)$

$v'(4) = 2\cos\left(\frac{4^2}{2}\right)(4)$

$v'(4) = -1.164$

Slowing down because $v(4)$ is positive and $v'(4)$ is negative.

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$1 + 2\sin\left(\frac{t^2}{2}\right) = 0$

$t = 2.707$

The particle changes direction one time at $t = 2.707$ because $v(t) = 0$ and $v(t)$ changes from positive to negative.

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(c) Find the position of the particle at time $t = 0$.

$2 + \int_4^0 v(t)\,dt = \boxed{-3.815}$

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

$\int_0^3 |v(t)|\,dt = \boxed{5.301}$

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2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

a(t) & v(t) same or diff sign?

$v(4) = 1 + 2\sin\left(\frac{4^2}{2}\right) = 2.979$ +

$a(t) = 2\cos\left(\frac{t^2}{2}\right) \cdot t =$

$a(4) = -1.164$ −

particle is slowing down at t=4 b/c a(t) & v(t) have different signs

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$v(t) = 0$

$1 + 2\sin\left(\frac{t^2}{2}\right) = 0$

$t = 2.707468$

particle changes direction at t=2.707 because the velocity changes sign at that time

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(c) Find the position of the particle at time $t = 0$.

position: $\int v(t)\,dt$

$p = \int 1 + 2\sin\left(\frac{t^2}{2}\right) dt$ $\quad u = \frac{1}{2}t^2 \quad du = t\,dt$

$\frac{1}{t}du = dt$

$pos = \frac{1}{t}\int 1 + 2\sin(u)\,du$

$\frac{1}{t}\left(t - 2\cos u\right) + c$

$\frac{1}{t}\left(t - 2\cos\left(\frac{t^2}{2}\right)\right)$

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

total distance $= \int_0^3 |v(t)|\,dt$

$\int_0^3 \left|1 + 2\sin\left(\frac{t^2}{2}\right)\right| dt = \boxed{5.301}$

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2C

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

At time t=4 the particle is slowing down

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right) = 0$

$v'(t) = t + 2t^2\cos\left(\frac{t^2}{2}\right) = 0$

particle changes direction at t = 2.607 and t = 1.375

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(c) Find the position of the particle at time $t = 0$.

$$\int_4^0 \left(1+2\sin\left(\frac{t^2}{2}\right)\right) dt$$

$$= -\int_0^4 \left(1+2\sin\left(\frac{t^2}{2}\right)\right) dt$$

$$= -5.815$$

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

$$\int_0^3 \left|\left(1+2\sin\left(\frac{t^2}{2}\right)\right)\right| dt$$

$$= 5.301$$

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Question 2

Overview

In this problem students were given information about a particle moving along the x-axis for time $t \geq 0$. The velocity of the particle is given as a trigonometric function, and the particle is at position $x = 2$ at time $t = 4$. In part (a) students needed to conclude that the particle is slowing down at $t = 4$ because $v(4)$ and $v'(4)$ have different signs. In part (b) students needed to determine when the particle changes direction in the interval $0 < t < 3$, and justify their answer. This required use of the calculator to solve $v(t) = 0$ on $0 < t < 3$. In part (c) students needed to apply the Fundamental Theorem of Calculus to find the position of the particle at time $t = 0$; i.e., $x(0) = x(4) - \int_0^4 v(t)\,dt$. The expression is evaluated using the calculator. In part (d) students needed to find the total distance the particle travels from $t = 0$ to $t = 3$. Students were expected to set up and evaluate $\int_0^3 |v(t)|\,dt$ (or an appropriate sum of definite integrals) using the calculator.

Sample: 2A
Score: 9

The response earned all 9 points.

Sample: 2B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student's work is correct. The student is not required to explicitly state that $a(4) = v'(4)$. In part (b) the student's work is correct. In part (c) the student is not working with a definite integral and did not earn the first point. The student was not eligible to earn the other 2 points. In part (d) the student's work is correct.

Sample: 2C
Score: 3

The response earned 3 points: no points in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student has a conclusion without a reason, so no points were earned. In part (b) the student reports two incorrect values of t. The student did not earn the first point and was not eligible for the second point. In part (c) the student earned the first point for a correct definite integral. The student does not use the initial condition and was not eligible to earn the other 2 points. In part (d) the student's work is correct.