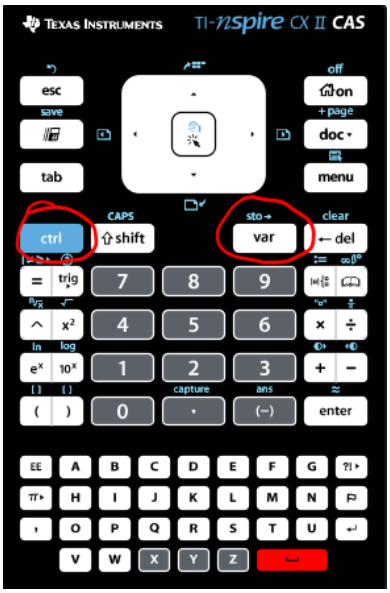
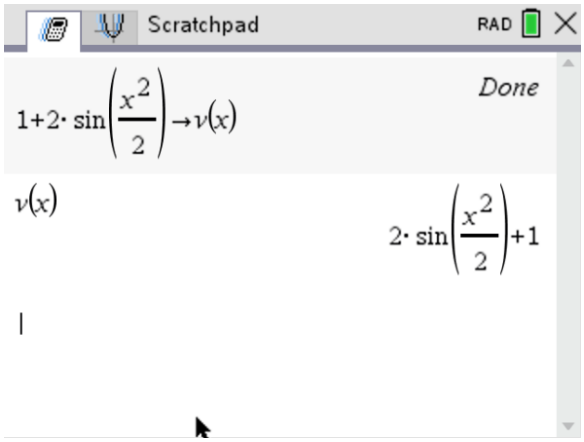
2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

Similar to other calculator problems, I would store this function into my calculator. (If you cannot store into your calculator, that is okay)





* I used x instead of t, and if you do that too, be sure to remember to put answers with t.

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(a) At time $t = 4$, is the particle speeding up or slowing down?

In order to determine if a particle is speeding up or slowing down, we need to compare velocity to acceleration.

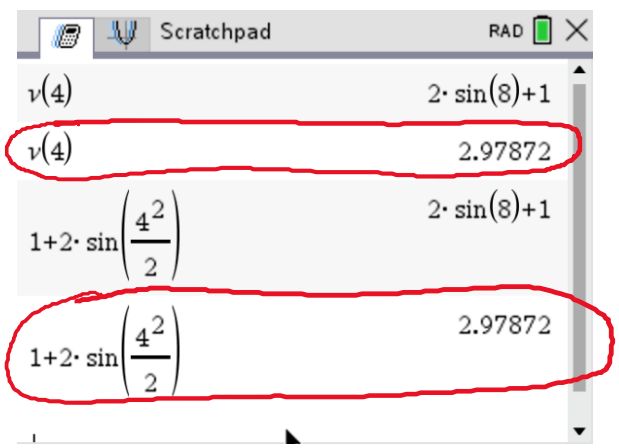
If the signs of velocity and acceleration are the same (both positive or both negative), the particle is <u>speeding up</u>.

→ Speeding up in the positive direction or negative direction.

If the signs of velocity and acceleration are opposite, the particle is <u>slowing down</u>.

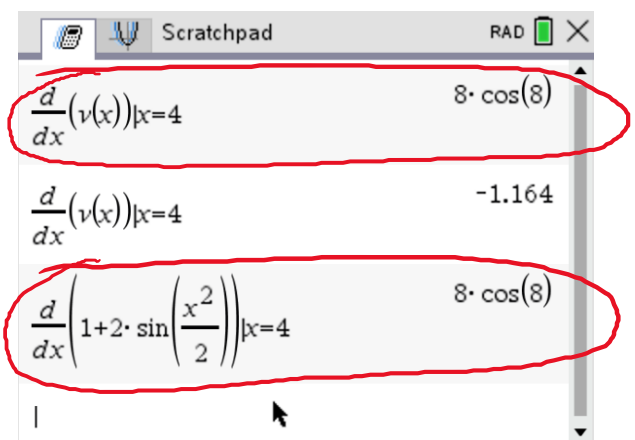
$v(4) > 0$

Notice, you still get the same answer even if you do not / cannot store the function.



$a(4) = v'(4) < 0$

Remember, acceleration is the derivative of velocity. Again, same answer.



$\boxed{\text{Because } v(4) > 0 \text{ and } a(4) < 0, \text{ the particle is slowing down at } t = 4.}$

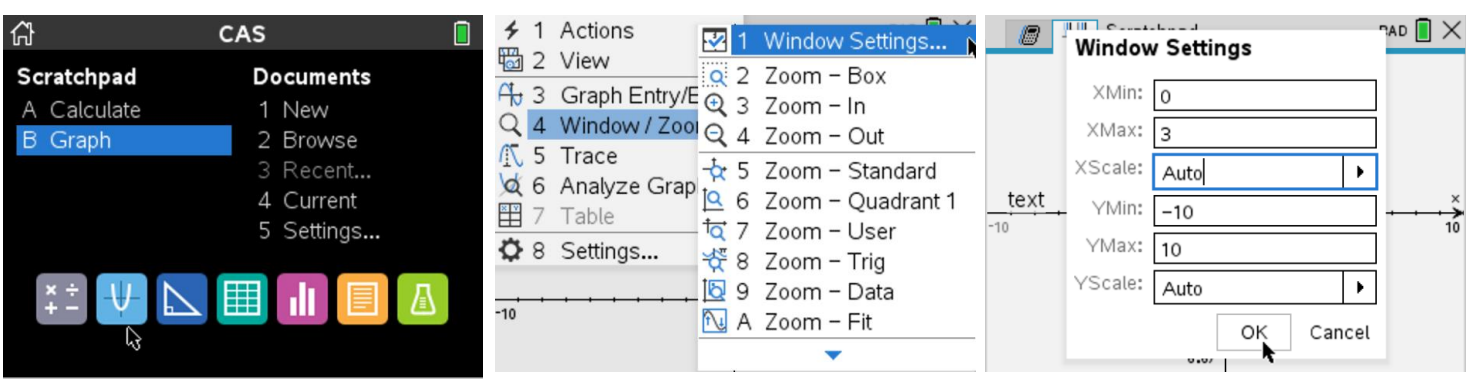
2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

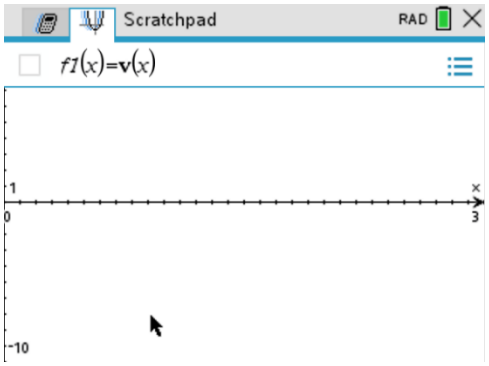
(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

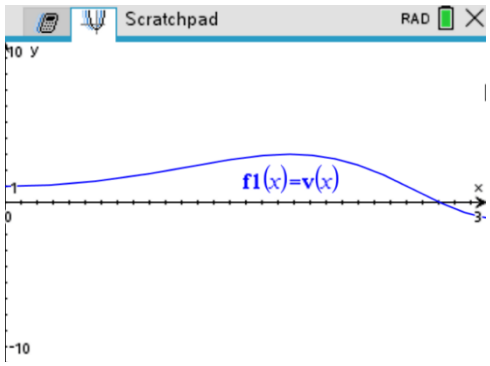
In order for the particle to change direction, $v(t)$ of the particle must change signs (either changing from moving in a positive direction to moving in a negative direction or vice versa).

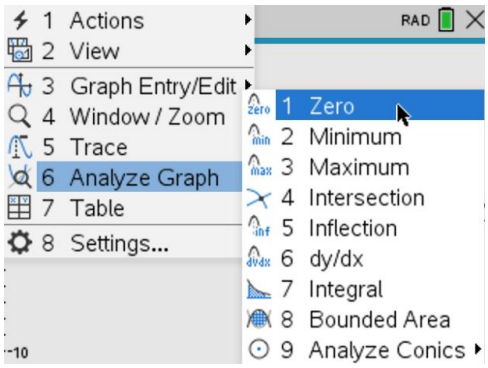
In order for $v(t)$ to change signs, it must equal to 0. So, we need to find the critical number(s).

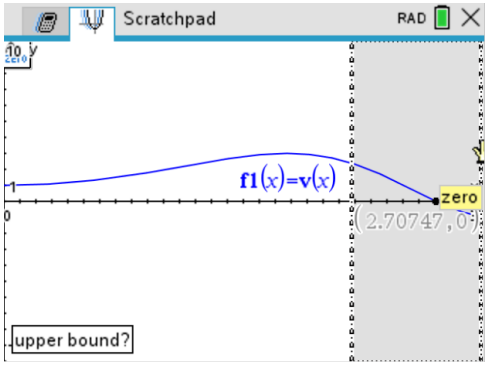
When is $v(t) = 0$? Since $v(t)$ is a sine function, it may be easier to graph and set our window to fit the interval $0 < t < 3$.

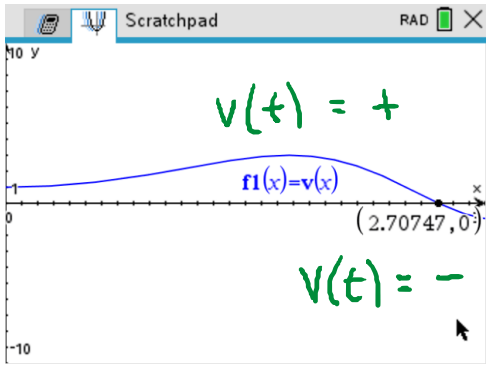












Critical number is $t = 2.707$.

$v(t)$ changes from positive to negative at $t = 2.707$, $\therefore$ the particle changes direction.

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(c) Find the position of the particle at time $t = 0$.

Since we are given velocity, in order to find the position of the particle, we need to take the integral of $v(t)$.

Now, the integral of $v(t)$ gives us the displacement of the particle, so in order to determine any specific position, we must be given an initial position

(which we are at $t=4$, $x=2$).

$$x(0) = x(4) + \int_4^0 v(t)\,dt$$

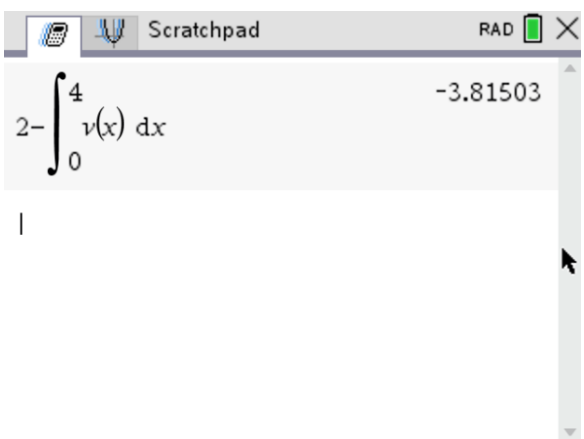
Position we want

Position we know

Displacement from what we know to what we want

$$x(0) = x(4) - \int_0^4 v(t)\,dt$$

$$x(0) = 2 - \int_0^4 v(t)\,dt$$



$$\boxed{x(0) \approx -3.815}$$

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(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

To find the total distance traveled when given velocity, we need to take the integral. However, in determining total

distance traveled, it does not matter if the particle traveled in a positive or negative direction, it all gets counted as positive. ∴ we need to use absolute value of v(t) to make all of the velocity values positive.

$$\boxed{\int_0^3 |v(t)|\,dt \approx 5.301}$$