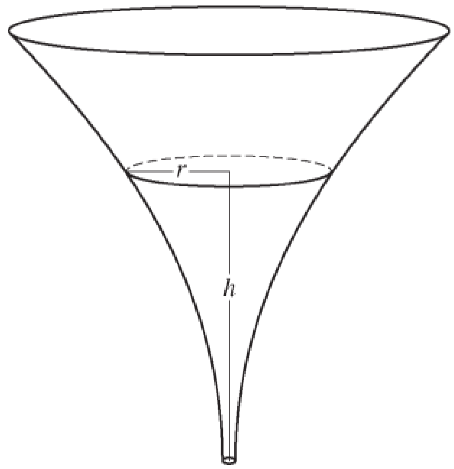


5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

Remember, average value is found by $\frac{\text{Integral}}{\text{Interval}}$. Also, we are tasked to find the average value of the radius and we are given the equation of the radius.

$$\frac{\int_0^{10} \frac{1}{20}(3+h^2)\,dh}{10-0} \Rightarrow \frac{1}{10}\cdot\frac{1}{20}\int_0^{10}(3+h^2)\,dh$$

$$\Rightarrow \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$$

$$\Rightarrow \frac{1}{200}\left[\left(3(10) + \frac{(10)^3}{3}\right) - \cancel{\left(3(0) + \frac{(0)^3}{3}\right)}\right]$$

$$\Rightarrow \frac{1}{200}\left[30 + \frac{1000}{3}\right]$$

← Could leave answer here on AP Test. Just be sure to include units.

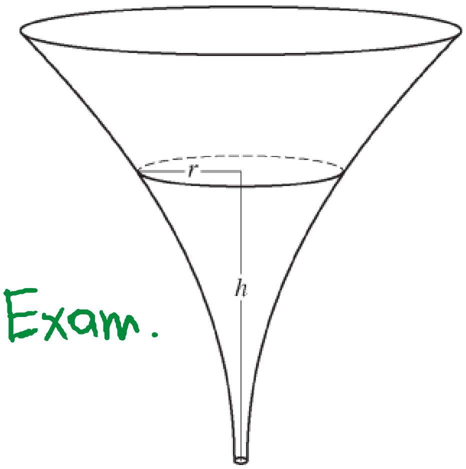
$$\Rightarrow \frac{1}{200}\left[\frac{90}{3} + \frac{1000}{3}\right]$$

$$\Rightarrow \frac{1}{20\cancel{0}}\left[\frac{109\cancel{0}}{3}\right]$$

$$= \boxed{\frac{109}{60} \text{ inches}}$$

← This represents the average length of the radius throughout the funnel from a height of $0 \le h \le 10$.

* Volume will not be tested on 2020 AP AB Exam.



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.

(b) Find the volume of the funnel.

This funnel is made of circular cross sections and remember, the 3D volume is the total of all the 2D areas. To find a total, we need to take an integral of the area of a circle, which is $A = \pi r^2$, from $0 \le h \le 10$.

$$\int_0^{10} \pi \left[\frac{1}{20}(3+h^2)\right]^2 dh$$

$u = 3 + h^2$

$du = 2h\, dh$

$dh = \frac{du}{2h}$

$$\int_0^{10} \pi \cdot \frac{1}{400} \cdot (3+h^2)^2 \cdot dh$$

u-substitution does not work

$$\frac{\pi}{400}\int_0^{10}(3+h^2)^2\,dh \qquad 9+6h^2+h^4$$

$$\uparrow$$

$$(3+h^2)^2=(3+h^2)(3+h^2)=9+3h^2+3h^2+h^4$$

$$\frac{\pi}{400}\int_0^{10}(9+6h^2+h^4)\,dh$$

$$\frac{\pi}{400}\left[9h+2h^3+\frac{h^5}{5}\right]_0^{10}$$

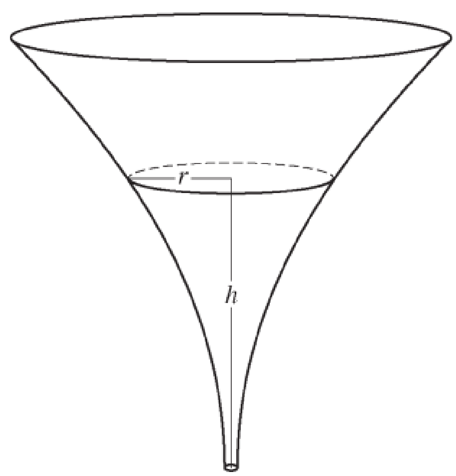
$$\frac{\pi}{400}\left[\left(9(10)+2(10)^3+\frac{(10)^5}{5}\right)-\cancel{\left(9(0)+2(0)^3+\frac{(0)^5}{5}\right)}\right]$$

$$\frac{\pi}{400}\left[90+2(1000)+\frac{100000}{5}\right]$$

$$\frac{\pi}{400}\left[90+2000+20000\right]$$

$\frac{\pi}{40\cancel{0}}\left[2209\cancel{0}\right]$

$\boxed{\frac{2209\pi}{40} \text{ cubic inches}}$



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

This is a related rates problem, so we need to determine the equation we need to use, the information we need to use, and to take the derivative of our equation.

When: $h = 3$

$$\frac{dr}{dt} = -\frac{1}{5} \quad \text{(decreasing)}$$

Find: $\frac{dh}{dt} = ?$

Since we are only using these three parts, we can use the equation of the radius given to us (instead of using a new or different equation).

$$\frac{d}{dt}\left[r = \frac{1}{20}\left(3 + h^2\right)\right]$$

Remember, "when" information must be plugged in after taking derivative.

$$\frac{dr}{dt} = \frac{1}{\cancel{20}_{10}}\left(\cancel{2}h\frac{dh}{dt}\right)$$

$$\frac{dr}{dt} = \frac{1}{10}h\frac{dh}{dt}$$

$$-\frac{1}{5} = \frac{1}{10}(3)\frac{dh}{dt}$$

$$\frac{\cancel{10}^{2}}{3} \cdot -\frac{1}{\cancel{5}_{1}} = \frac{3}{10}\frac{dh}{dt} \cdot \frac{10}{3}$$

$$\boxed{\frac{dh}{dt} = -\frac{2}{3} \text{ in/sec}}$$

Rate of Change is measured in units per time.