

$$g(3) = 6$$

$$f(6) = 4$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

$$g'(3) = 2$$

$$f'(6) = 5$$

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

Recall that the 2 things needed for an equation of the line tangent is:

1. Point @ $x = 3$ and 2. Slope @ $x = 3$

To find point at $x = 3$, substitute into our function: $k(3) = f(g(3))$

$$= f(6)$$

$$k(3) = 4$$

To find slope at $x = 3$, take the derivative of $k(x)$ using the Chain Rule and then substitute $x = 3$ into the resulting $k'(x)$.

$$k(x) = f(g(x)) \rightarrow k'(3) = f'(6) \cdot 2$$

$$k'(x) = f'(g(x)) \cdot g'(x)$$

$$k'(3) = f'(g(3)) \cdot g'(3)$$

$$k'(3) = 5 \cdot 2$$

$$k'(3) = 10$$

$$y - 4 = 10(x - 3) \quad \leftarrow \text{Equation for tangent line to graph of } k \text{ at } x = 3.$$

↑

Leave in this form.

$$f(1) = -6$$

$$g'(1) = 8$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

$$g(1) = 2$$

$$f'(1) = 3$$

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

In this problem, we need to find the derivative of $h(x)$ using the Quotient Rule and then substitute $x=1$ into our $h'(x)$ function.

$$h'(x) = \frac{f(x) \cdot g'(x) - g(x) \cdot f'(x)}{[f(x)]^2}$$

$$h'(1) = \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{[f(1)]^2}$$

$$h'(1) = \frac{(-6) \cdot (8) - (2) \cdot (3)}{(-6)^2}$$

$$h'(1) = \frac{-48 - 6}{36}$$

$$h'(1) = \frac{-54}{36} \Rightarrow \boxed{h'(1) = \frac{-3}{2}}$$

$$f'(6) = 5$$

$$f'(2) = -2$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(c) Evaluate $\int_1^3 f''(2x) dx$.

In order to take the integral in this problem, we need to use u-substitution.

$$u = 2x$$

$$\int_1^3 f''(u) \frac{du}{2}$$

$$du = 2 dx$$

$$dx = \frac{du}{2}$$

$$\frac{1}{2} \int_1^3 f''(u) du$$

$$u(3) = z(3) = 6 \quad \text{and} \quad u(1) = z(1) = 2$$

$$\frac{1}{2} \int_2^6 f''(u) du$$

I chose to change
the interval to
be in terms of u .

$$\frac{1}{2} \left[f'(u) \right]_2^6$$

$$\frac{1}{2} \left[f'(6) - f'(2) \right]$$

$$\frac{1}{2} [5 - (-2)]$$

$$\frac{1}{2} [7]$$

$$\boxed{\int_1^3 f''(2x) dx = \frac{7}{2}}$$