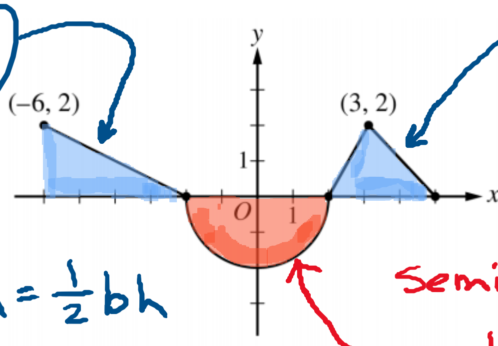
$A = \frac{1}{2}(4)(2) = 4$

$A = \frac{1}{2}(3)(2) = 3$



Graph of f'

Triangle: $A = \frac{1}{2}bh$

Semicircle: $A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(2)^2 = 2\pi$

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

This problem has tasked us to find two different values for f and we are given the graph of f'.
f is the antiderivative of f' (or the area under a curve when given a graph).
So, we need to use an integral and we cannot forget about our given condition of $f(-2) = 7$.

$$f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx$$

What we want

What we know

The displacement from what we know to what we want

$$f(-6) = f(-2) - \int_{-6}^{-2} f'(x)\,dx$$

$$f(-6) = 7 - [4]$$

$$\boxed{f(-6) = 3}$$

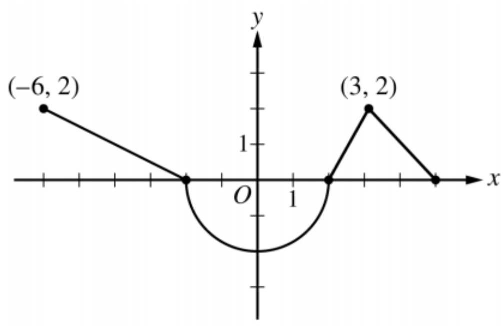
$$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx$$

Same Setup

Remember, area under the x-axis is negative for integrals.

$$f(5) = 7 + [-2\pi + 3]$$

$$\boxed{f(5) = 10 - 2\pi}$$



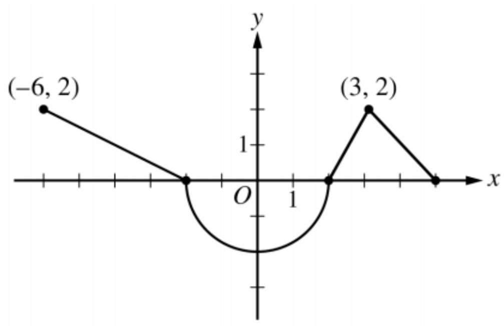
Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(b) On what intervals is f increasing? Justify your answer.

Recall, f is increasing when its derivative, f', is positive. Since we are given the graph of f', we need to look at when the graph of f' is above the x-axis (meaning f' has positive values).

f' has positive values on $-6 \le x < -2$ and $2 < x < 5$, $\therefore$ f is increasing.



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

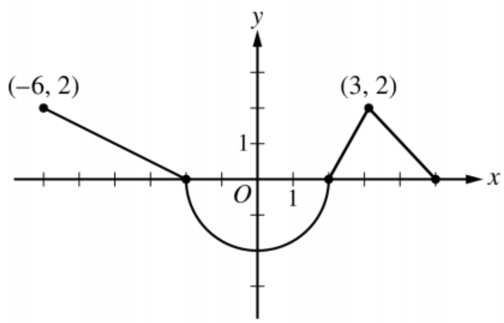
(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

Maximum and minimum values for f occur when f' is zero or does not exist. But, the keyword here is absolute, so we must also consider the endpoints of $x=-6$ and $x=5$ (which we already determined in part (a) above). And remember, we are given the graph of f'.

$7-2\pi < 3$

x	f(x)
-6	3
-2	$f(-2) = f(-2) + \int_{-2}^{-2} f'(x)\,dx \Rightarrow 7 + 0 = 7$
2	$f(2) = f(-2) + \int_{-2}^{2} f'(x)\,dx \Rightarrow 7 - 2\pi = 7 - 2\pi$
5	$10 - 2\pi$ $(2\pi < 7 \therefore 10 - 2\pi > 3)$

The abolute minimum for f is $f(2) = 7 - 2\pi$. (Problem asks for the actual minimum value, not when it occurs)



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

f'' is the derivative of f'. Since we are given the graph of f', we need only look at the slope of the graph for $x = -5$ and $x = 3$.

$f'(-5)$ is on a linear segment that has a $\frac{\text{rise}}{\text{run}}$ of $\frac{-2}{4}$ or $-\frac{1}{2}$ $\therefore$ $\boxed{f''(-5) = -\frac{1}{2}}$

At $f'(3)$ there is a corner, meaning $\lim_{x \to 3^-} f''(x) \neq \lim_{x \to 3^+} f''(x)$. $\boxed{f''(3) \Rightarrow \text{DNE}}$

Slope from Left ≠ Slope from Right