- This was one of the two most difficult problems in the last 10 years.
  - Out of the 300,000+ students that took the AP Test in 2017 worldwide, 49% received no points at all.
  - But, it is just like any other problem we've done, only with different notation.

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

The two things needed for an equation of a tangent line at $t=0$ are:

1. Point @ $t=0$ and 2. Slope @ $t=0$

$H(0) = 91$

$\frac{dH}{dt} = -\frac{1}{4}(H-27)$

The only difference here is that we have H in our differential equation instead of t.

$= -\frac{1}{4}(91-27)$

$= -\frac{1}{4}(64)$

$H'(0) = -16$

Tangent Line Equation:

$y - 91 = -16(t-0)$

$y - 91 = -16t$

$y = -16t + 91$

Remember, you can use any of these.

To approximate $H(t)$ @ $t=3$, we simply just substitute $t=3$ into our tangent line equation and solve for y.

$$y = -16(3) + 91$$

$$y = -48 + 9$$

$$y = 43$$

The internal temperature of the potato at $t=3$ is approximately 43 degrees.

Any of these can be used as your answer here

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   (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

Recall, the 2^{nd} Derivative Test tells us if our function has a max or min at a critical number (1^{st} derivative used to find).

If the 2^{nd} derivative is positive, that means our function is concave up.

which means there is a minimum (or underestimate for our problem).

If the 2nd derivative is negative, that means our function is concave down, which means there is a maximum (or overestimate for our problem).

$$\frac{dH}{dt} = -\frac{1}{4}(H-27)$$

We are taking the derivative of the 1st derivative with respect to t.

$$\frac{d}{dt}\left[\frac{dH}{dt} = -\frac{1}{4}H + \frac{27}{4}\right]$$

$$\frac{d^2H}{dt^2} = -\frac{1}{4}\cdot\frac{dH}{dt}$$

Derivative of H with respect to t.

$$\frac{d^2H}{dt^2} = -\frac{1}{4}\left[-\frac{1}{4}(\underline{H-27})\right]$$

Substitute for $\frac{dH}{dt}$

We are told in the problem that $H(t) > 27$ always.

$$\frac{d^2H}{dt^2} = -\frac{1}{4}\left[-\frac{1}{4}(+)\right] = +$$

Positive

Since $\frac{d^2H}{dt^2}$ is positive, $H(t)$ is concave up, meaning our answer in part (a) is an underestimate approximation.

Good news, this whole part (b) was only worth one point.

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   (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

For this part, we are finding $G(t)$ given $\frac{dG}{dt}$. This means we must take the integral.

$$\frac{dG}{(G-27)^{2/3}} = -dt \leftarrow$$ Separate variables before taking integral

$$\int (G-27)^{-2/3}\, dG = \int -dt$$ May need to use u-substitution

$u = G - 27$

$du = dG$

$$\int u^{-2/3}\, du = \int -\, dt$$

$$\frac{u^{1/3}}{1/3} = -t + C$$

$$3(G-27)^{1/3} = -t + C$$

$$3(91-27)^{1/3} = -0 + C$$

Substitute given condition of $G(0) = 91$

$$3\sqrt[3]{64} = C$$

$$3(4) = C = 12$$

$$3(G-27)^{1/3} = -t + 12$$

Now, solve for G.

$$(G-27)^{1/3} = \frac{-t+12}{3}$$

$$G - 27 = \left(\frac{-t+12}{3}\right)^3$$

$$G(t) = \left(\frac{-t+12}{3}\right)^3 + 27 \quad \text{for} \quad 0 < t < 10$$

$$G(3) = \left(\frac{-3+12}{3}\right)^3 + 27$$

$$= \left(\frac{9}{3}\right)^3 + 27$$

$$= 3^3 + 27$$

$$= 27 + 27$$

$$G(3) = 54$$

The internal temperature of the potato at $t=3$ is 54°C.