5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

The particles are moving on the x-axis. Which means if the velocity is negative, the particle is moving left and if the velocity is positive, the particle is moving right.

Also, for the particle P, we are given the position function. So, we need to take the derivative to get the velocity.

The derivative of $\ln u$ is $\frac{u'}{u}$.

$u = t^2 - 2t + 10 \Rightarrow u' = 2t - 2$

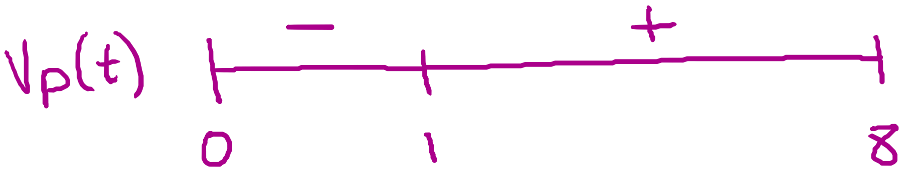
$$\frac{d}{dt}\left[\ln\left(t^2 - 2t + 10\right)\right] \Rightarrow v_p(t) = \frac{2t-2}{t^2-2t+10}$$

Next, we need to determine when the velocity is zero or does not exist. So, set both the numerator and denominator equal to 0.

$2t - 2 = 0$
$2(t-1) = 0$
$t = 1$

$t^2 - 2t + 10 = 0$
Does not equal 0

$$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)} \rightarrow \sqrt{-36}$$



Choose a test value within each interval to substitute into $V_P(t)$.

Since $V_P(t)$, or the velocity of particle P, is negative on $0 < t < 1$ the particle is moving to the left.

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5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

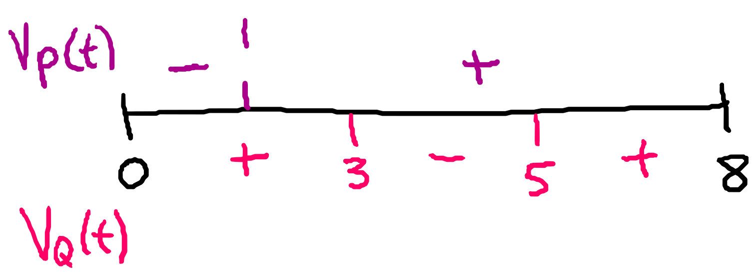
   (b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

For this part, we already found the direction of particle P. Now, we just need to find the direction of particle Q and compare. We are already given the velocity function for particle Q.

$$0 = t^2 - 8t + 15$$

$$0 = (t-5)(t-3)$$

$$t = 5 \text{ and } t = 3$$



Particles P and Q have the same sign for their velocity on $1 < t < 3$ and $5 < t < 8$.

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

Recall, acceleration is the derivative of velocity and to determine if a particle's speed is increasing, decreasing, or neither, we need to compare the signs of the velocity and acceleration.

$$V_Q(t) = t^2 - 8t + 15 \Rightarrow a_Q(t) = 2t - 8$$

$$V_Q(2) = (2)^2 - 8(2) + 15 \qquad a_Q(2) = 2(2) - 8$$

$$= 4 - 16 + 15 \qquad = 4 - 8$$

$$V_Q(2) = 3 \qquad a_Q(2) = -4$$

Since the velocity and acceleration of particle Q have different signs at $t = 2$, then the speed of particle Q is decreasing.

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(d) Find the position of particle Q the first time it changes direction.

There are 2 things to pay attention to here:

1. We need to find the position of Q, but have the velocity function of Q. ∴ we need to take the integral; but to know the particle's exact position, we are given an initial condition of $t=0$, $x=5$.
2. We need the 1st time Q changes direction, so if we look back at part (b), we see that Q changes direction at $t=3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt$$

Position on the x-axis that we want to know.

Position on the x-axis that we are given.

Displacement of Q on the x-axis from where we know to where we want to go.

$$x_Q(3) = 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_0^3$$

$$= 5 + \left[\left(\frac{1}{3}(3)^3 - 4(3)^2 + 15(3)\right) - \cancel{\left(\frac{1}{3}(0)^3 - 4(0)^2 + 15(0)\right)}\right]$$

$$= 5 + \frac{1}{3}(27) - 4(9) + 45$$

$$= 5 + 9 - 36 + 45$$

$$x_Q(3) = 23$$