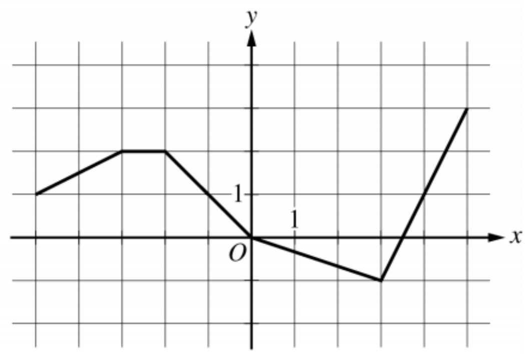
x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

We are given 3 different functions, but in part (a) we only need to use f(x). Also, we only need to find the slope (or derivative) and not the entire equation of a tangent line.

$f(x) = \cos(2x) + e^{\sin x}$

Recall your derivative rules and remember to use Chain Rule.

$f'(x) = -\sin(2x) \cdot 2 + e^{\sin x} \cdot \cos x$

Next, we can substitute $x = \pi$ and solve.

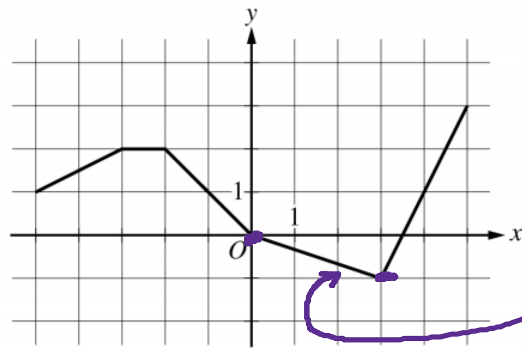
$f'(\pi) = -\sin(2\pi) \cdot 2 + e^{\sin \pi} \cdot \cos \pi$

$= \cancel{0 \cdot 2} + e^0 \cdot (-1)$

$$\boxed{f'(\pi) = -1}$$

---

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

$h'(2) = -\frac{1}{3}$

$\frac{\text{Rise}}{\text{Run}}$

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

In part (b), we are using $h(x)$ and $f(x)$. Remember, for $h(x)$ we simply need to look at the slope of the graph.

$k(x) = h(f(x))$ ← Again, make sure to use Chain Rule.

$k'(x) = h'(f(x)) \cdot f'(x)$

Next, to find $k'(\pi)$ we substitute $x = \pi$ and remember that $f'(\pi) = -1$ from part (a).

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi)$

→ $f(\pi) = \cos(2\pi) + e^{\sin \pi} \Rightarrow 1 + e^0 \Rightarrow 1 + 1 = 2$

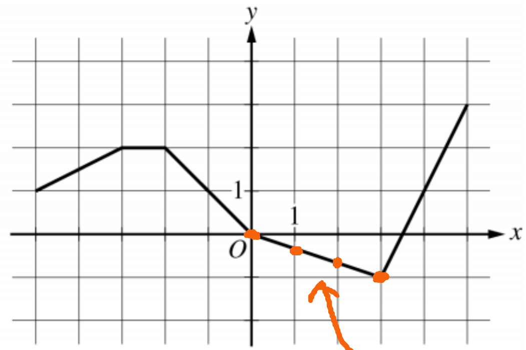
$k'(\pi) = h'(2)\cdot(-1)$

$= \left(-\frac{1}{3}\right)\cdot(-1)$

$\boxed{k'(\pi) = \frac{1}{3}}$

---

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

$h(0) = 0$

$h(1) = -\frac{1}{3}$

$h(2) = -\frac{2}{3}$

$h(3) = -1$

Slope $: -\frac{1}{3}$

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(c) Let m be the function defined by $m(x) = g(-2x)\cdot h(x)$. Find $m'(2)$.

In part (c), we need to use $g(x)$ and $h(x)$. Remember, we must use Product Rule and Chain Rule. Also, the derivatives for $g(x)$ are given to us in the table.

$m(x) = g(-2x)\cdot h(x)$

Product Rule

Chain Rule

$m'(x) = g(-2x)\cdot h'(x) + h(x)\cdot g'(-2x)\cdot(-2)$

Next, we substitute $x = 2$ and solve.

$$m'(2) = g(-2\cdot 2)\cdot h'(2) + h(2)\cdot g'(-2\cdot 2)\cdot(-2)$$

$$m'(2) = g(-4)\cdot h'(2) + h(2)\cdot g'(-4)\cdot(-2)$$

Find the value in the table.

Use from part (b).

Find the value on the graph.

Find the value in the table.

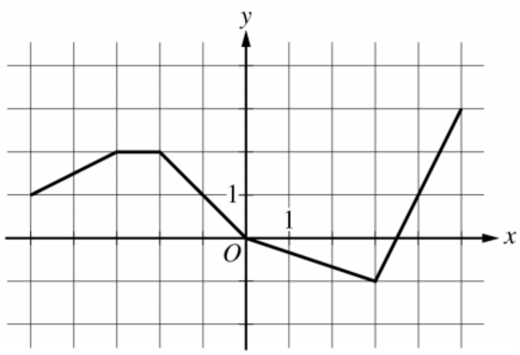
$$m'(2) = (5)\cdot\left(-\tfrac{1}{3}\right) + \left(-\tfrac{2}{3}\right)\cdot(-1)\cdot(-2)$$

$$m'(2) = \left(-\frac{5}{3}\right) + \left(-\frac{4}{3}\right)$$

$$m'(2) = -\frac{9}{3} \Rightarrow \boxed{m'(2) = -3}$$

---

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

   Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

   Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

   (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

The setup of this question leads us to think about one of our theorems. Firstly, $g' = -4$ is not given on our table, but the table does not list every number on $[-5,-3]$. Secondly, $g' = -4$ is an instantaneous rate of change (IROC), so we need to get our mind to thinking about IROC = AROC.

AROC $$\frac{g(-3)-g(-5)}{(-3)-(-5)} = \frac{2-10}{-3+5} = \frac{-8}{2} = -4$$

Since $g(x)$ is differentiable and $\therefore$ continuous, and the average rate of change of $g(x)$ on $[-5,-3]$ is -4, by MVT (mean value theorem) there must be some x value such that $g'(x) = -4$.