AP® CALCULUS AB
2018 SCORING GUIDELINES

Question 5

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is $\dfrac{f(\pi)-f(0)}{\pi-0} = \dfrac{-e^{\pi}-1}{\pi}$.

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$f'\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}\cos\left(\dfrac{3\pi}{2}\right) - e^{3\pi/2}\sin\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}$

The slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$ is $e^{3\pi/2}$.

2 : { 1 : $f'(x)$; 1 : slope }

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$

x	$f(x)$
0	1
$\dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}e^{\pi/4}$
$\dfrac{5\pi}{4}$	$-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$.

3 : { 1 : sets $f'(x) = 0$; 1 : identifies $x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$ as candidates ; 1 : answer with justification }

(d) $\lim\limits_{x\to\pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim\limits_{x\to\pi/2} g(x) = g\left(\dfrac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

$\lim\limits_{x\to\pi/2} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to\pi/2} \dfrac{f'(x)}{g'(x)} = \dfrac{-e^{\pi/2}}{2}$.

3 : { 1 : g is continuous at $x = \dfrac{\pi}{2}$ and limits equal 0 ; 1 : applies L'Hospital's Rule ; 1 : answer }

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives