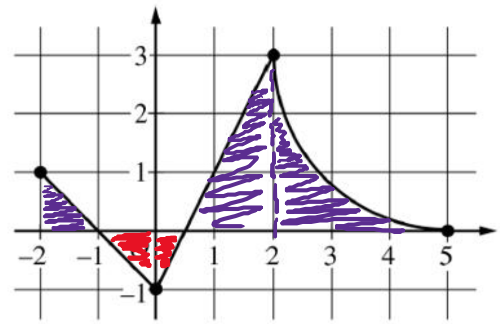


Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

Remember, to find an integral given a graph, we find the area under the curve.

$$\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$$

$\int_{-2}^{0} f(x)\,dx = 0 \Rightarrow$ These two triangles cancel each other out.

$\int_{0}^{1/2} f(x)\,dx \overset{\text{Triangle}}{=} \frac{1}{2}\left(\frac{1}{2}\right)(1) = \frac{1}{4} = -\frac{1}{4} \Rightarrow$ Below the x-axis.

$\int_{1/2}^{3} f(x)\,dx \overset{\text{Triangle}}{=} \frac{1}{2}\left(\frac{3}{2}\right)(3) = \frac{9}{4} \Rightarrow$ Above the x-axis.

$\int_{3}^{5} f(x)\,dx =$ This one involves a little bit more work...

Square − Quartercirle

$$\int_3^5 f(x)\,dx = 3(3) - \frac{1}{4}\pi(3)^2$$

$$\int_3^5 f(x)\,dx = 9 - \frac{9}{4}\pi$$

$$\int_{-2}^5 f(x)\,dx =$$

$$\int_{-2}^0 f(x)\,dx + \int_0^{1/2} f(x)\,dx + \int_{1/2}^3 f(x)\,dx + \int_3^5 f(x)\,dx$$

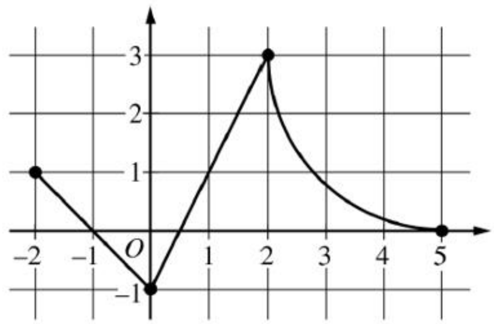
$$= 0 + \left(-\frac{1}{4}\right) + \frac{9}{4} + \left(9 - \frac{9}{4}\pi\right)$$

$$= 2\cancel{\frac{8}{4}} + 9 - \frac{9}{4}\pi = 11 - \frac{9}{4}\pi$$

$$\int_{-6}^5 f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^5 f(x)\,dx$$

$$7 = \int_{-6}^{-2} f(x)\,dx + 11 - \frac{9}{4}\pi$$

$$\boxed{\int_{-6}^{-2} f(x)\,dx = -4 + \frac{9}{4}\pi}$$



Graph of f

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(b) Evaluate $\int_3^5 (2f'(x) + 4)\,dx$.

Because of the addition sign, we can split this integral into:

$$\int_3^5 (2f'(x)+4)\,dx = \int_3^5 2f'(x)\,dx + \int_3^5 4\,dx$$

Factor the constant out

$$= 2\int_3^5 f'(x)\,dx + \int_3^5 4\,dx$$

The integral of $f'(x)$ is $f(x)$.

$$= 2\Big[f(x)\Big]_3^5 + \Big[4x\Big]_3^5$$

The integral of 4 is $4x$.

Fundamental Theorem of Calculus.

$$= 2\Big[f(5) - f(3)\Big] + \Big[4(5) - 4(3)\Big]$$

$= 2\left[0-(3-\sqrt{5})\right] + \left[20-12\right]$

Use the graph & given info

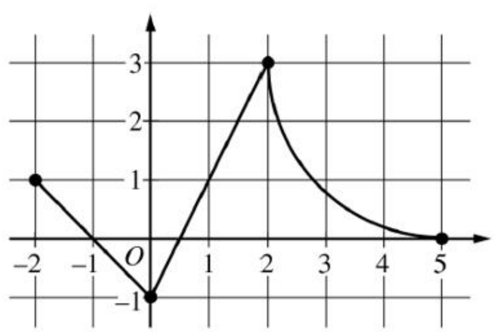
On the AP Test, I would leave my answer right here unsimplified.

* Be careful that the constant of 2 only goes with f(x) and not the 4x.

$= 2\left[-3+\sqrt{5}\right] + \left[8\right]$

Simplified answer

$= -6 + 2\sqrt{5} + 8 = \boxed{2+2\sqrt{5}}$



Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

In order to find a max or min, we need

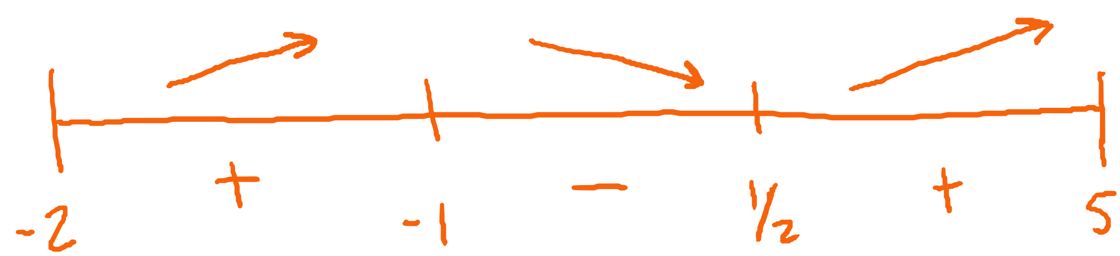
to determine when the derivative, $g'(x)$, is equal to 0 or ONE.

$$g(x) = \int_{-2}^{x} f(t)\,dt$$

$$g'(x) = f(x)$$

Looking at the graph, $f(x) = 0$ at $x = -1$ and $x = \frac{1}{2}$; $f(x)$ is a continuous function from $-2 \le x \le 5$ $\therefore$ exists on all points on the interval.

$g'(x) = f(x)$



Looking at our number line, $g(x)$ can only have an absolute maximum on $[-2, 5]$ at either $x = -1$ or $x = 5$ due to the slopes or $g'(x)$ or $f(x)$.

$g(-1) = \int_{-2}^{-1} f(x)\,dx = \frac{1}{2}(1)(1) = \frac{1}{2}$

Use area under the curve for the graph.

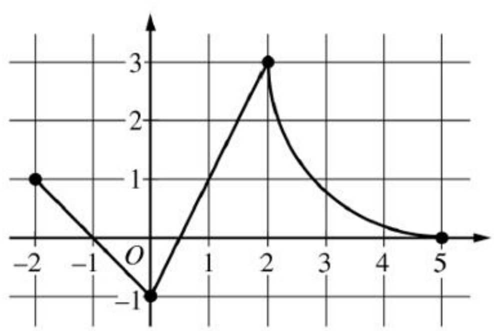
$g(5) = \int_{-2}^{5} f(x)\,dx = 11 - \frac{9}{4}\pi$

Use from part (a)

$11 - (2.25)(3.14)$

This number is greater than $\frac{1}{2}$.

Absolute maximum value for $g(x)$ on $-2 \le x \le 5$ is $11 - \frac{9}{4}\pi$ at $x = 5$.



Graph of f

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(d) Find $\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.

Seeing this limit problem, your mind may go straight to L'Hôpital's Rule, but remember, the 1st step to solving a limit is to substitute the x-value.

$$\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan(1)}$$

Look at the slope of the graph at $x = 1$ to determine $f'(1)$. $f'(1) = 2$

Look at the value of the graph at $x = 1$ to determine $f(1)$. $f(1) = 1$

For arctan(1), you need to the think of when does tan = 1? $\tan x = 1$

$\arctan(1) = \frac{\pi}{4}$ $\qquad x = \frac{\pi}{4}$

$$\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10 - 3(2)}{1 - \frac{\pi}{4}}$$

For AP Test, leave answer here.