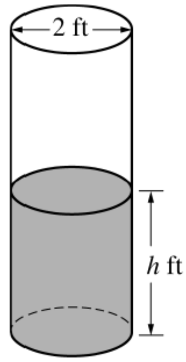


4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\boxed{\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

Rate of change is another term for derivative. Take the derivative with respect to t and substitute the "when" information after taking the derivative. Remember that constant information can be substituted before taking derivative. The radius of a cylinder will not change.

$$V = \pi r^2 h \Rightarrow V = \pi (1)^2 h$$

$$\frac{d}{dt}\left[V = \pi h\right] \Rightarrow \frac{dV}{dt} = \pi \frac{dh}{dt}$$

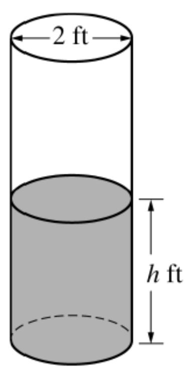
Derivative with respect to t and π is a constant.

$$\frac{dV}{dt} = \pi\left(-\frac{1}{10}\sqrt{h}\right)$$

h=4 is the "when" information.

$$\boxed{\frac{dV}{dt} = \pi\left(-\frac{1}{10}\sqrt{4}\right) \text{ ft}^3/\text{sec}}$$

Leave answer here on AP Test.
Make sure to include correct units, which is the rate of change of volume (ft³) over time (seconds).



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(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

The wording of this problem is a little tricky, but it is asking if the derivative of h is increasing or decreasing. So, we want to know if $\frac{dh}{dt}$ is increasing or decreasing. $\therefore$ we need to take the derivative of $\frac{dh}{dt}$.

$$\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$$

$$\frac{d}{dt}\left[\frac{dh}{dt} = -\frac{1}{10}h^{1/2}\right]$$

Take the derivative with respect to t.

$$\frac{d^2h}{dt^2} = -\frac{1}{20}h^{-1/2}\cdot\frac{dh}{dt}$$

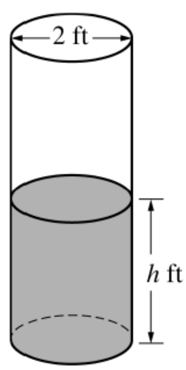
$$\frac{d^2h}{dt^2} = -\frac{1}{20\cancel{\sqrt{h}}}\cdot\left(-\frac{1}{10}\cancel{\sqrt{h}}\right)$$

$$\frac{d^2h}{dt^2} = \frac{1}{200}$$

Because $\frac{d^2h}{dt^2}$ is always positive, that means that $\frac{dh}{dt}$ is always increasing.

Another way to think about this is:

$\frac{dh}{dt}$ is the velocity and it is always negative (meaning the cylinder is losing water); $\frac{d^2h}{dt^2}$ is the acceleration and it is always positive (meaning the cylinder is losing water at a more rapid pace). Velocity is increasing in the negative direction.



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(c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

We know the differential equation for h, but not an actual equation for h. So, to get this equation, we need to take the integral.

$$\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$$

$$\frac{1}{\sqrt{h}}\,dh = -\frac{1}{10}\,dt$$

$$\int h^{-1/2}\,dh = \int -\frac{1}{10}\,dt$$

$$\frac{h^{1/2}}{1/2} = -\frac{1}{10}t + C$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10}(0) + C$$

$$2\sqrt{5} = C$$

$$\frac{1}{2}\left(2\sqrt{h}\right) = \left(-\frac{1}{10}t + 2\sqrt{5}\right)\frac{1}{2}$$

$$\left(\sqrt{h}\right)^2 = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$$

$$\boxed{h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2}$$