

Graph of  $f$

3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined

by 
$$g(x) = \int_2^x f(t) dt.$$

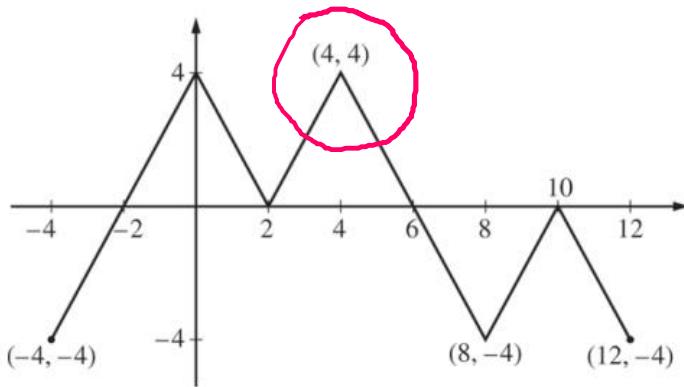
- (a) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 10$ ? Justify your answer.

In order for  $g$  to have a max or min at  $x=10$ ,  $g'$  needs to be equal to 0 and the sign of  $g'$  must change.

$g'(x) = f(x)$  and our graph is of  $f(x)$  ∵ it is of  $g'(x)$ .

$g'(x) = 0$  at  $x=10$ , but  $g'(x)$  does not change signs (The values of the graph are negative on either side of  $x=10$ ).

The answer is neither.



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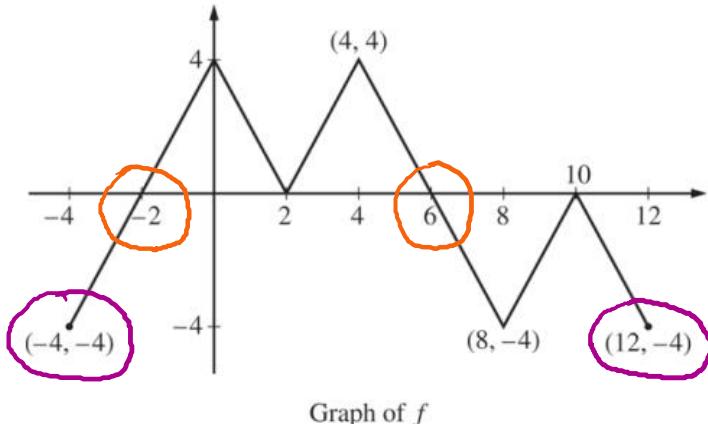
3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined by  $g(x) = \int_2^x f(t) dt$ .

(b) Does the graph of  $g$  have a point of inflection at  $x = 4$ ? Justify your answer.

In order for  $g$  to have a point of inflection at  $x=4$ ,  $g''$  needs to be equal to 0 or DNE and the sign of  $g''$  must change.  $g''(x) = f'(x)$  which is the slope of graph given.

At  $x=4$ , the slope (derivative) ONE because there is a corner. The slope changes at  $x=4$ , meaning the sign of  $g''(x)$  changes.

$\therefore g$  has a point of inflection at  $x = 4$ .



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- (c) Find the absolute minimum value and the absolute maximum value of  $g$  on the interval  $-4 \leq x \leq 12$ . Justify your answers.

Remember, in order for there to be a critical number, the derivative needs to be equal to 0 or DNE and the sign of the derivative must change.

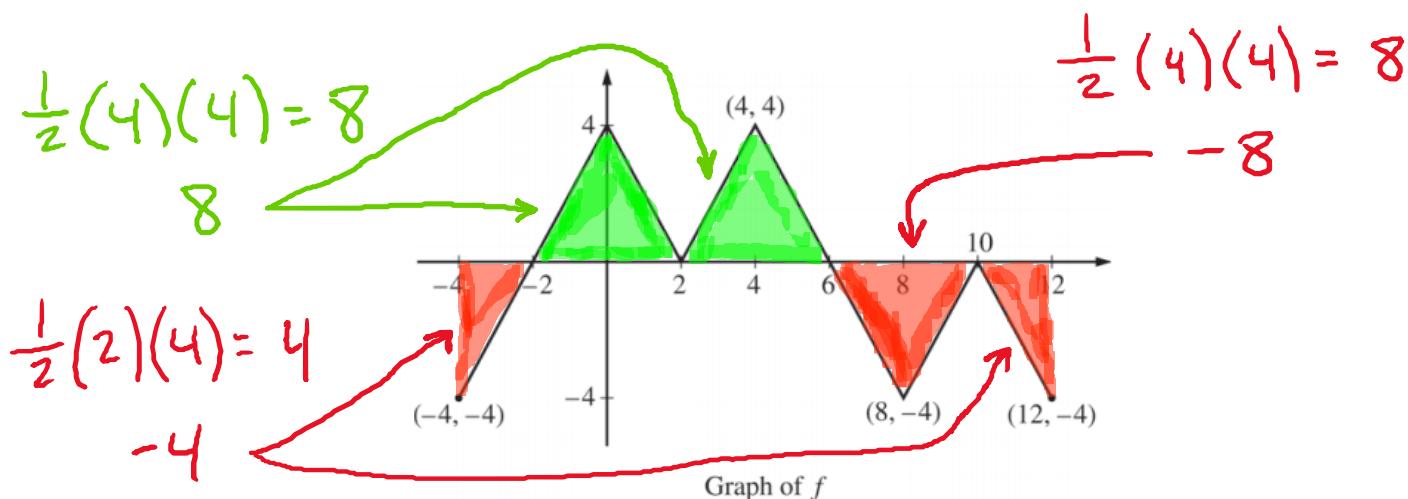
The only  $x$ -values that are considered critical numbers are  $x = -2$  and  $x = 6$ .  
(Remembering  $g'(x) = f(x)$  ... our graph.)

Since this problem wants absolute max and min, we need to consider endpoints of  $x = -4$  and  $x = 12$ .

This problem also wants us to find the actual values... which is  $y$ .

To find these  $y$ -values, we need to substitute  $x = -4$ ,  $x = -2$ ,  $x = 6$ , and  $x = 12$  into  $g(x) = \int_2^x f(t) dt$ .

Remember, to find an integral using a graph, we need to use the area under the curve.



$$\begin{aligned} g(-4) &= \int_{-4}^{-2} f(x) dx \\ &= - \int_{-4}^2 f(x) dx \\ &= - [-4 + 8] \end{aligned}$$

$$\begin{aligned} g(-2) &= \int_{-2}^2 f(x) dx \\ &= - \int_{-2}^2 f(x) dx \\ &= - [8] \end{aligned}$$

$$g(-4) = -4$$

$$g(-2) = -8$$

$$g(6) = \int_2^6 f(x) dx$$

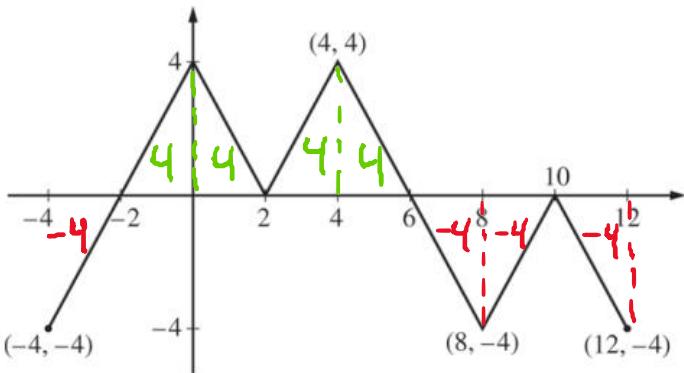
$$g(6) = 8$$

$$g(12) = \int_2^{12} f(x) dx$$

$$= [8 - 8 - 4]$$

$$g(12) = -4$$

On the interval  $-4 \leq x \leq 12$ , the maximum value for  $g$  is 8 and the minimum value is -8.



Graph of  $f$

3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined by  $g(x) = \int_2^x f(t) dt$ .

- (d) For  $-4 \leq x \leq 12$ , find all intervals for which  $g(x) \leq 0$ .

To find the values for  $g(x)$ , we need to use the values we found in part (c) and the area under the curve.

$x$	-4	-2	0	2	4	6	8	10	12
$g(x)$	-4	-8	-4	0	4	8	4	0	-4