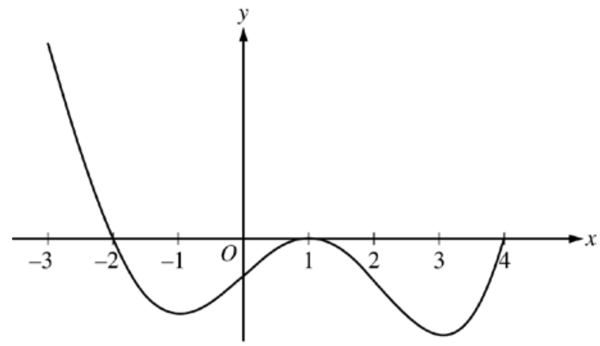
Question 5



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

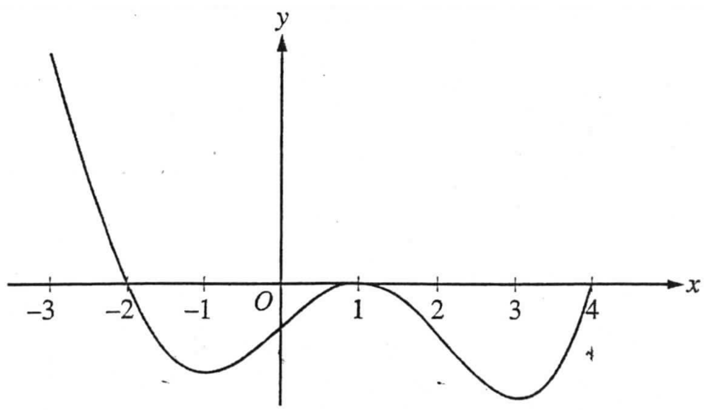
(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$. $f'(x)$ changes from positive to negative at $x = -2$. Therefore, f has a relative maximum at $x = -2$.	$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$
(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$
(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points. The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.	$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{and } 3 \\ 1 : \text{reason} \end{cases}$
(d) $f(x) = 3 + \int_1^x f'(t)\,dt$ $f(4) = 3 + \int_1^4 f'(t)\,dt = 3 + (-12) = -9$ $f(-2) = 3 + \int_1^{-2} f'(t)\,dt = 3 - \int_{-2}^1 f'(t)\,dt$ $= 3 - (-9) = 12$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

1&2

5A1



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

$f(x)$ has a relative maximum at $x = -2$ because $f'(x)$ switches from positive to negative ~~at~~ this point.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

The graph of f is both concave down and decreasing on the intervals $(-2, -1)$ and $(1, 3)$ because on these intevals $f'(x)$ is negative and also $f''(x)$ is negative.

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5A$_2$

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

$x = -1, 1, 3$

The x-coordinates of the points of inflection for the graph of f are $x=-1$, $x=1$, $x=3$. This is because at these points, $f''(x)$ switches signs.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$$\boxed{f(x) = \int_1^x f'(t)\,dt + 3}$$

$$f(4) = \int_1^{(4)} f'(t)\,dt + 3$$

$$= (-12) + 3$$

$$\boxed{f(4) = -9}$$

$$f(-2) = \int_1^{(-2)} f'(t)\,dt + 3$$

$$= (9) + 3$$

$$\boxed{f(-2) = 12}$$

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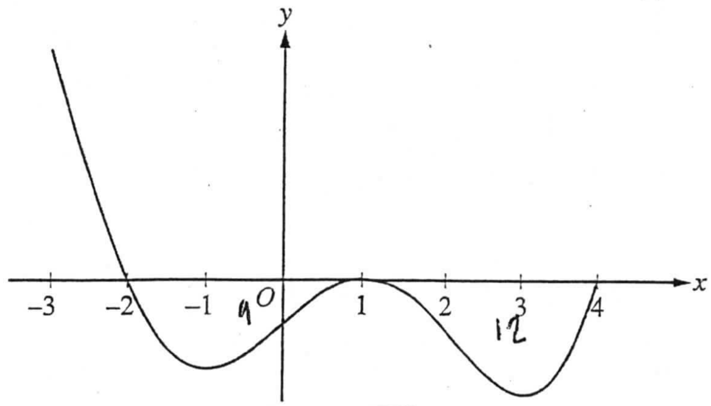
5B.



Graph of $\boxed{f'}$

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(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

at $x=-2$, f has a relative max because this is where $f'(x)$ changes from positive to negative.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

$(-2,-1)$ and $(1,3)$ because this is where $f'(x)$ is negative and the slope of $f'(x)$ is negative.

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5B₂

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(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

the graph of f has points of inflection at $x=-1$, $x=1$, and $x=3$ because this is where $f'(x)$ has a slope of 0.

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(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$f(x) = \int f'(x)\,dx$

$f(4) = \int_1^4 f'(x)\,dx = -12$

$f(-2) = \int_{-2}^{1} f'(x)\,dx = -9$

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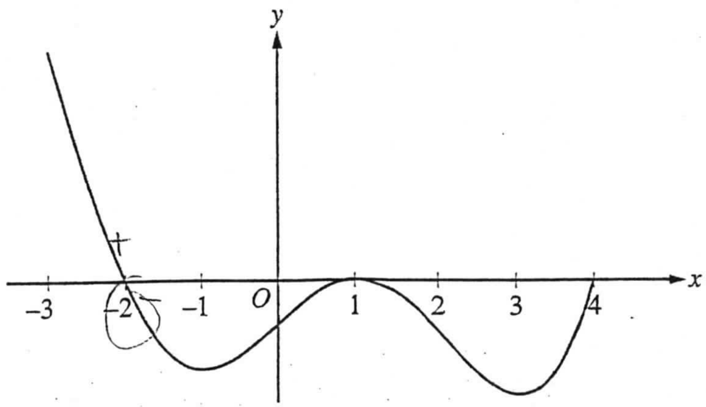

Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

f has a relative maximum at point -2 because the graph of f' at -2 goes from positive to negative.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

It is concave down at point -2 because it has a relative maximum at that point.

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NO CALCULATOR ALLOWED

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

It has a point of inflection at point 1 because at that point it equals zero but it never passes the x-axis.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$$\int_1^4 f'(x)dx = f(4) - f(1)$$

$$\int_1^4 f'(x)dx = f(4) - 3$$

$$+3 \qquad +3$$

$$f(4) = \int_1^4 f'(x)dx + 3$$

$$\int_{-2}^1 f'(x)dx = f(1) - f(-2)$$

$$\int_{-2}^1 f'(x)dx = 3 - f(-2)$$

$$f(-2) = 3 - \int_{-2}^1 f'(x)dx$$

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Question 5

Overview

In this problem students were given the graph of f', the derivative of a twice-differentiable function f on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ were given. In part (a) students had to find all x-coordinates at which f has a relative maximum. Students had to find the critical points $x = -2$, $x = 1$, and $x = 4$ from the graph of f' and apply the First Derivative Test to conclude that the relative maximum occurs at $x = -2$. In part (b) students were asked to determine the open intervals where the graph of f is both concave down and decreasing. Students needed to use the graph of f' to determine the open intervals where f' was both decreasing and negative in order to answer the question. In part (c) students were asked to find the x-coordinates of all points of inflection for the graph of f. Students needed to use the graph of f' to determine the x-coordinates of the points where f' changes from increasing to decreasing or from decreasing to increasing in order to answer the question. In part (d) students were asked to write an expression for $f(x)$ that involves an integral given that $f(1) = 3$. Students were expected to use the Fundamental Theorem of Calculus to produce $f(x) = 3 + \int_1^x f'(t)\,dt$. Students had to use properties of the definite integral, including the relationship of the definite integral to the areas of the bounded regions to find $f(4)$ and $f(-2)$.

Sample: 5A
Score: 9

The response earned all 9 points.

Sample: 5B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student correctly identifies the x-coordinates of the points of inflection, so the first point was earned. The student's reason is not sufficient to earn the second point since "$f'(x)$ has a slope of 0" does not, in general, guarantee a point of inflection. In part (d) the student earned the first point for having $f'(x)$ as the integrand in a definite integral. The student does not provide an expression for $f(x)$ nor calculate correct values for $f(4)$ and $f(-2)$, so the second and third points were not earned.

Sample: 5C
Score: 3

The response earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not identify any intervals. In part (c) the student identifies only one of the three correct values, so the first point was not earned. The student's reason is not sufficient to earn the second point. In part (d) the student earned the first point for having $f'(x)$ as the integrand in a definite integral. The student does not provide an expression for $f(x)$ nor calculate correct values for $f(4)$ and $f(-2)$, so the second and third points were not earned.