6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

Remember, two things are needed to write a tangent line:

1. Point and
2. Slope

The point $(-1, 1)$ is given to us, and $\frac{dy}{dx}$ is a notation for slope.

Substitute $(-1,1)$ into $\frac{dy}{dx} = \frac{y}{3y^2 - x}$

and get $\frac{dy}{dx} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

Write the tangent line in point-slope form and leave it in that form.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{4}(x + 1)$$

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

A vertical tangent line occurs when the derivative (or slope) is undefined.

A rational function is undefined when its denominator equals 0.

$3y^2 - x = 0 \Rightarrow 3y^2 = x$

$y^3 - (3y^2)y = 2$

$y^3 - 3y^3 = 2$

$-2y^3 = 2$

$y^3 = -1$

$y = -1$

$3(-1)^2 = x$

$3 = x$

The coordinate for a vertical tangent line to the curve is $(3, -1)$.

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

$\frac{d^2y}{dx^2}$ is the derivative of $\frac{dy}{dx}$ and to take the derivative of $\frac{dy}{dx} = \frac{y}{3y^2 - x}$, we need to use the Quotient Rule with respect to x.

$$\frac{(3y^2 - x)\left(\frac{dy}{dx}\right) - (y)\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

We want to find $\frac{d^2y}{dx^2}$ at $(-1, 1)$ and we found $\frac{dy}{dx}$ at $(-1, 1)$ in part (a), which was $\frac{1}{4}$.

Now, substitute $x = -1$, $y = 1$, and $\frac{dy}{dx} = \frac{1}{4}$ into the $\frac{d^2y}{dx^2}$.

$$\frac{(3(1)^2 - (-1))\left(\frac{1}{4}\right) - (1)\left(6(1)\cdot\frac{1}{4} - 1\right)}{(3(1)^2 - (-1))^2}$$

$$= \frac{4\left(\frac{1}{4}\right) - \left(\frac{3}{2} - 1\right)}{(4)^2}$$

$$= \frac{1 - \frac{1}{2}}{16}$$

$$= \frac{\frac{1}{2}}{16}$$

$$= \frac{1}{2} \cdot \frac{1}{16}$$

$$= \frac{1}{32}$$